Lehrstuhl und Institut für Nachrichtentechnik
RWTH AACHEN UNIVERSITY

Lösung zur Klausur
Grundgebiete der Elektrotechnik 3

Univ.-Prof. Dr.-Ing. Jens-Rainer Ohm

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09:00h

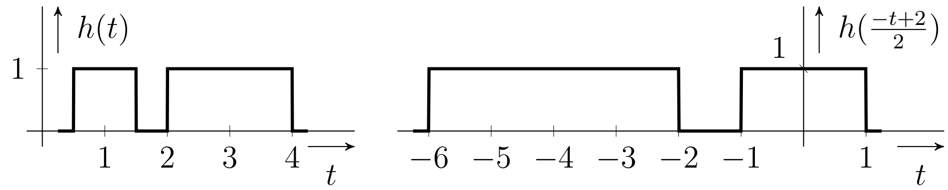
Erlaubte Hilfsmittel:
Formelsammlung Grundgebiete Elektrotechnik 3, eine leere Folie

3. September 2020

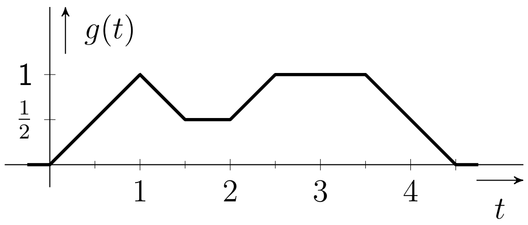
Lösung zu Aufgabe 1 H2020

1.1 $\frac{1}{2} \leq T \leq 3.$

1.2



1.3



1.4

$$h_\varepsilon(t) = h(t) * \varepsilon(t) = \begin{cases} 0 & t \leq \frac{1}{2}, \\ \int\limits_{\frac{1}{2}}^{t} 1 \,\mathrm{d}\tau = t - \dfrac{1}{2} & \frac{1}{2} < t \leq \frac{3}{2}, \\ \int\limits_{\frac{1}{2}}^{\frac{3}{2}} 1 \,\mathrm{d}\tau = 1 & \frac{3}{2} < t \leq 2, \\ \int\limits_{\frac{1}{2}}^{\frac{3}{2}} 1 \,\mathrm{d}\tau + \int\limits_{2}^{t} 1 \,\mathrm{d}\tau = t - 1 & 2 < t \leq 4, \\ \int\limits_{\frac{1}{2}}^{\frac{3}{2}} 1 \,\mathrm{d}\tau + \int\limits_{2}^{4} 1 \,\mathrm{d}\tau = 3 & 4 < t. \end{cases}$$

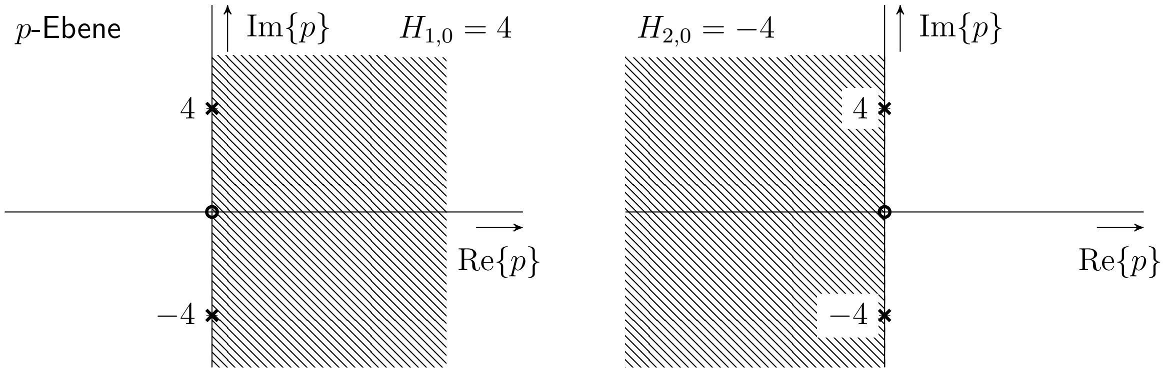
1.5 $h_1(t) = \varepsilon(t-1) + \varepsilon(t-2)$

1.6 Das System ist kausal, da $h(t) = 0$ für $t < 0$.

Das System ist nicht stabil, da $\int\limits_{-\infty}^{+\infty} |h(t)| \,\mathrm{d}t \not< \infty$.

2.1 $H_1(p) = \dfrac{4p}{p^2+16}$; $\quad H_2(p) = -\dfrac{4p}{p^2+16}$

2.2



2.3 Es existiert keine Laplace-Transformierte von $h(t)$, da kein gemeinsamer Konvergenzbereich von $H_1(p)$ und $H_2(p)$ existiert.

2.4

$$H_3(p) = A_0 + \frac{A_1}{p+\mathrm{j}} + \frac{A_2}{p-\mathrm{j}} + \frac{A_3}{p+2\mathrm{j}} + \frac{A_4}{p-2\mathrm{j}}$$
$$A_0 = 0,\ A_1 = \mathrm{j},\ A_2 = -\mathrm{j},\ A_3 = \mathrm{j}\frac{3}{4},\ A_4 = -\mathrm{j}\frac{3}{4}$$

Alternativ:

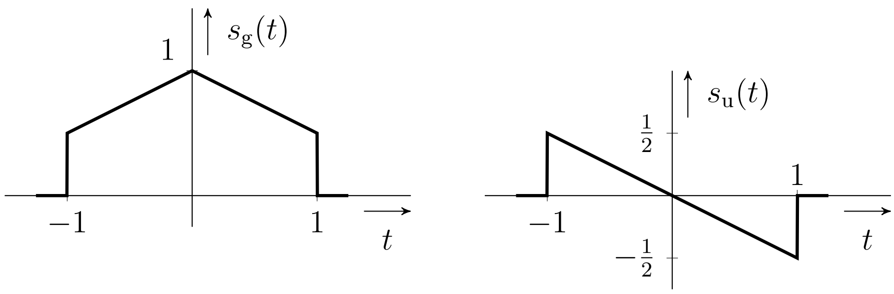
$$H_3(p) = \frac{Ap+B}{p^2+1} + \frac{Cp+D}{p^2+4} + E$$
$$A = C = 0,\ B = 2,\ D = 3,\ E = 0$$

2.5 $h_3(t) = \mathrm{j}\left[\mathrm{e}^{-\mathrm{j}t} - \mathrm{e}^{\mathrm{j}t} + \frac{3}{4}\mathrm{e}^{-\mathrm{j}2t} - \frac{3}{4}\mathrm{e}^{\mathrm{j}2t}\right]\varepsilon(t) = \left[2\sin(t) + \frac{3}{2}\sin(2t)\right]\varepsilon(t)$

2.6 Nicht stabil, da die imaginäre Achse nicht im Konvergenzbereich liegt.

2.7 $h_4(t) = \mathcal{L}^{-1}\{p \cdot H_3(p)\} = \frac{\mathrm{d}}{\mathrm{d}t}h_3(t) = [2\cos(t) + 3\cos(2t)]\,\varepsilon(t)$

3.1



3.2 $S_u(f) = -\int\limits_{-1}^{1} \frac{t}{2} e^{-j2\pi f t} \, dt = -\frac{j}{2\pi f} \left[\cos(2\pi f) - si(2\pi f)\right]$

3.3

$$S_1(f) = 2\, si^2(2\pi f) e^{-j4\pi f}$$

$$E_{s_1} = 2 \int\limits_0^2 \left(\frac{t}{2}\right)^2 dt = \frac{4}{3}$$

3.4

$$k = 0: \quad S_{1,p}(0) = \frac{1}{T} \int\limits_0^T s_1(t)\, dt = \frac{1}{4}$$

$$k \neq 0: \quad S_{1,p}(k) = F S_1(f = kF) = \frac{1}{8} S_1\left(\frac{k}{8}\right) = \frac{1}{4} si^2\left(\frac{2\pi k}{8}\right) e^{-j\frac{4\pi k}{8}} = \frac{1}{4} si^2\left(\frac{\pi k}{4}\right) (-j)^k$$

3.5 Innerhalb des Filters liegen nur die Koeffizienten für $k = \pm 2$

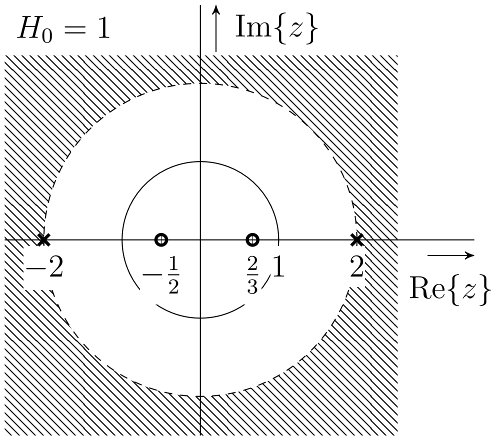
$$\begin{aligned} G_{1,p}(f) &= S_{1,p}(f) \cdot H(f) = S_{1,p}(2)\delta\left(f - \frac{1}{4}\right) + S_{1,p}(-2)\delta\left(f + \frac{1}{4}\right) \\ &= \frac{1}{4} si^2\left(\frac{\pi}{2}\right)\delta\left(f - \frac{1}{4}\right) + \frac{1}{4} si^2\left(-\frac{\pi}{2}\right)\delta\left(f + \frac{1}{4}\right) \\ &= -\frac{2}{\pi^2}\cos\left(\frac{\pi t}{2}\right) \end{aligned}$$

4.1

$$h(n) = 3^n \sin^2\left(\frac{\pi}{4}n\right)\varepsilon(n) = 3^n\frac{1}{2}\left[1-\cos\left(\frac{\pi}{2}n\right)\right]\varepsilon(n) = \frac{1}{2}3^n\varepsilon(n) - \frac{1}{2}3^n\cos\left(\frac{\pi}{2}n\right)\varepsilon(n)$$

$$H_1(z) = \frac{1}{2}\cdot\frac{1}{1-3z^{-1}} - \frac{1}{2}\cdot\frac{1-3\cos\left(\frac{\pi}{2}\right)z^{-1}}{1-6\cos\left(\frac{\pi}{2}\right)z^{-1}+9z^{-2}} = \frac{1}{2}\cdot\frac{1}{1-3z^{-1}} - \frac{1}{2}\cdot\frac{1}{1+9z^{-2}}$$

4.2

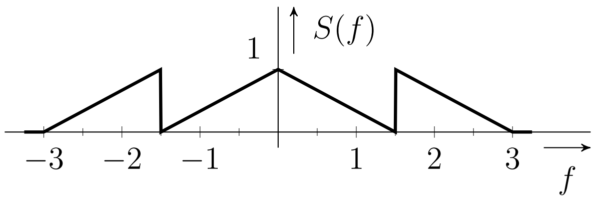


Nicht stabil, da der Einheitskreis nicht im Konvergenzbereich liegt.

4.3

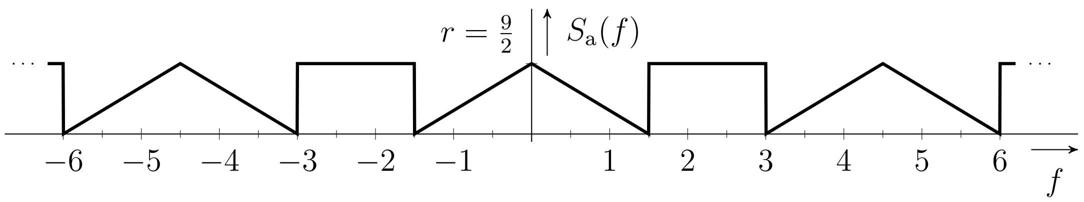
$$s(t) = 6\,\mathrm{si}^2(3\pi t) - 3\,\mathrm{si}(3\pi t)$$

$$S(f) = 2\Lambda\left(\frac{f}{3}\right) - \mathrm{rect}\left(\frac{f}{3}\right)$$



4.4 $f_\mathrm{g} = 3 \quad \Rightarrow r_\mathrm{min} = 2\cdot f_\mathrm{g} = 6$

4.5



4.6

$$G(f) = S_\mathrm{a}(f)\cdot H_\mathrm{TP}(f) = \frac{9}{2}\Lambda\left(\frac{f}{3/2}\right)\cdot\frac{2}{9}\mathrm{rect}\left(\frac{f}{3}\right) = \Lambda\left(\frac{2}{3}f\right)$$

$$\Rightarrow g(t) = \frac{3}{2}\,\mathrm{si}^2\left(\frac{3}{2}\pi t\right)$$

Lösung zu Aufgabe 5 **H2020**

5.1 $m_s^2 = \lim_{|\tau|\to\infty} \varphi_{ss}(\tau) = 1 \Rightarrow |m_s| = 1, \qquad L_s = \varphi_{ss}(0) = 3, \qquad \sigma_s^2 = L_s - m_s^2 = 2$

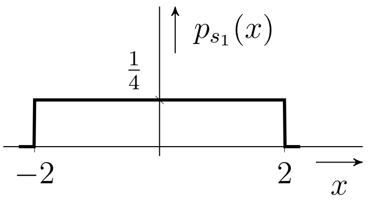
5.2

$$\begin{aligned}\mathcal{E}\left\{[s(t)+2s(t-2)]^2\right\} &= \mathcal{E}\left\{\left[s(t)^2 + 4s(t-2)^2 + 4s(t)s(t-2)\right]\right\}\\ &= \mathcal{E}\left\{s(t)^2\right\} + 4\,\mathcal{E}\left\{s(t-2)^2\right\} + 4\,\mathcal{E}\left\{s(t)s(t-2)\right\}\\ &= \varphi_{ss}(0) + 4\,\varphi_{ss}(0) + 4\,\varphi_{ss}(2) = 3 + 12 + 4 = 19\end{aligned}$$

5.3

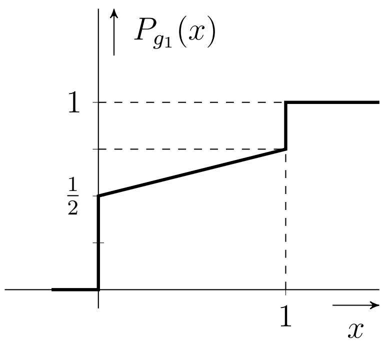
$$p_{s_1}(x) = \frac{1}{a}\,\mathrm{rect}\left(\frac{x - m_{s_1}}{a}\right) \qquad \text{mit } m_{s_1} = 0 \text{ und } \sigma_{s_1}^2 = L_{s_1} - m_{s_1}^2 = \frac{a^2}{12}$$

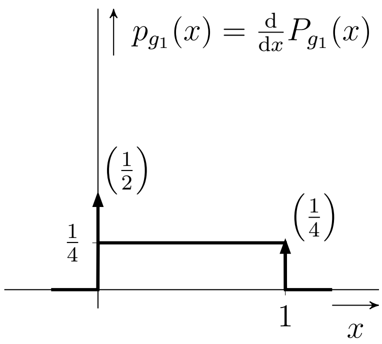
$$\Rightarrow a = 4 \qquad \Rightarrow p_{s_1}(x) = \frac{1}{4}\,\mathrm{rect}\left(\frac{x}{4}\right)$$



5.4

$$\begin{aligned}(x < 0) &: P_{g_1}(x) = \mathrm{Prob}\left[g_1(t) \le x\right] = 0\\ (0 \le x < 1) &: P_{g_1}(x) = \mathrm{Prob}\left[g_1(t) \le x\right] = \mathrm{Prob}\left[s_1(t) \le x\right] = P_{s_1}(x) = \frac{1}{2} + \frac{x}{4}\\ (x \ge 1) &: P_{g_1}(x) = \mathrm{Prob}\left[g_1(t) \le x\right] = 1\end{aligned}$$





5.5 $L_{g_1} = \int_{-\infty}^{\infty} x^2 p_{g_1}(x)\,\mathrm{d}x = 0 + \frac{1}{4}\int_0^1 x^2\,\mathrm{d}x + \frac{1}{4} = \frac{1}{12} + \frac{1}{4} = \frac{1}{3}$

5.6

$$\varphi_{s_2 s_2}(\tau) = \mathcal{F}^{-1}\left\{\phi_{s_2 s_2}(f)\right\} = \frac{N_0}{2}\delta(\tau)$$

$$\varphi_{s_2 g_2}(\tau) = \varphi_{s_2 s_2}(\tau) * h_2(\tau) = \frac{N_0}{2}\,\mathrm{rect}\left(\frac{\tau}{2} - \frac{1}{2}\right)$$

5.7

$$m_{s_2}^2 = \lim_{|\tau|\to\infty} \varphi_{s_2 s_2}(\tau) = 0 \Rightarrow m_{s_2} = 0$$

$$m_{g_2} = m_{s_2} H(0) = 0 \cdot H(0) = 0$$