Cheat sheet

SI-Units

- Base Units

Dimension	T	L	M	θ	N	I	I_v
Unit	s	m	kg	K	mol	A	cd

- Derived units

Hz	$\mathrm{s^{-1}}$
N	$\mathrm{kg\,m\,s^{-2}}$
Pa	$\mathrm{N\,m^{-2}}$
J	$\mathrm{N\,m}$
W	$\mathrm{J\,s^{-1}}$

Dimensionless numbers

Reynold number	Re	$\frac{v\,L}{\nu}$
Froude number	Fr	$\frac{v}{\sqrt{gh}}$
Shallowness parameter	ϵ	$\frac{H}{L}$
Peclet number	Pe	$\frac{Lv}{\alpha}$
Prandtl number	Pr	$\frac{\nu}{\alpha}$
Stefan number	Ste	$\frac{c_p \Delta T}{h_m}$ h_m : latent heat of melting
Strouhal number	Str	$\frac{fL}{v}$
Mach number	Ma	$\frac{v}{c}$
Lewis number	Le	$\frac{\alpha}{D}$

Miscellaneous

1. Shape factor

$$\alpha = \frac{\overline{u^2}}{\overline{u}^2}, \text{ where } \bar{*} = (1/h)\int_b^s * \, dz$$

2. Decomposition of the velocity profile

$$\mathbf{v}(\mathbf{x}+\mathbf{r}) = \mathbf{v}(\mathbf{x}) + \mathbf{w}\times\mathbf{r} + \mathbf{D}\cdot\mathbf{r}$$

$$\text{where } \mathbf{w} = \frac{1}{2}\nabla\times\mathbf{v}, \ \mathbf{D} = \frac{1}{2}\left(\nabla\mathbf{v}+\nabla\mathbf{v}^T\right), \ \mathbf{W} = \frac{1}{2}\left(\nabla\mathbf{v}-\nabla\mathbf{v}^T\right)$$

3. Cylindrical coordinate transformation rules

$$\begin{aligned}
\nabla f &= \frac{\partial f}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\vec{e}_\theta + \frac{\partial f}{\partial z}\vec{e}_z, \\
\nabla\cdot\vec{A} &= \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial\theta} + \frac{\partial A_z}{\partial z} \\
\nabla\times\vec{A} &= \left(\frac{1}{r}\frac{\partial A_z}{\partial\theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\vec{e}_z
\end{aligned}$$

4. Material derivative

$$\frac{D}{Dt}f = \partial_t f + \mathbf{v}\cdot\nabla f$$

5. Bernoulli equation

$$\tfrac{p}{\rho} + \tfrac{v^2}{2} + gz = const.alongastreamline$$

6. Error function

$$\operatorname{erf}(x) := \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}dy, \quad \operatorname{erfc}(x) := 1 - \operatorname{erf}(x)$$

$$\partial_x\operatorname{erf}(Cx) = \frac{2}{\sqrt{\pi}}Ce^{-(Cx)^2}, \quad \partial_x\operatorname{erfc}(Cx) = -\frac{2}{\sqrt{\pi}}Ce^{-(Cx)^2}$$

7. Thermal diffusivity

$$\alpha = \frac{\kappa}{\rho c_p}$$

Physical principles

1. Material symmetry

$$\hat{\sigma}^{(\zeta)}(*) = \hat{\sigma}^{(\eta)}(*P).$$

2. Material Isotropy

$$\hat{\sigma}(F) = \hat{\sigma}(V\cdot Q) = \hat{\sigma}(V\cdot Q\cdot P) = \hat{\sigma}(V\cdot Q\cdot Q^T) = \hat{\sigma}(V)$$

3. Material objectivity

$$\sigma^{(\mathbf{y})} = Q\cdot\sigma^{(\mathbf{x})}\cdot Q^T$$

4. Galilean invariance

$$\boldsymbol{\zeta} = \mathbf{x} - \mathbf{v}t$$

Mathematical models

1. Mass and momentum balance

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$
$$\partial_t(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho \mathbf{b}$$

2. Incompressible Euler

$$\nabla \cdot \mathbf{v} = 0$$
$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \mathbf{b}$$

3. Incompressible Navier-Stokes

$$\nabla \cdot \mathbf{v} = 0$$
$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \triangle \mathbf{v} + \mathbf{b}$$

4. Incompressible Navier-Stokes in cylindrical coordinates

$$\frac{\partial u_r}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right) u_r - \frac{u_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right)$$
$$\frac{\partial u_\theta}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right) u_\theta + \frac{u_r u_\theta}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + \nu \left(\nabla^2 u_\theta + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2} \right)$$
$$\frac{\partial u_z}{\partial t} + \left(\vec{u} \cdot \vec{\nabla}\right) u_z = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 u_z$$
$$\frac{1}{r} \frac{\partial}{\partial r} \left(r u_r \right) + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$

5. Heat equation

$$\partial_t T + \nabla \cdot (T \mathbf{v}) = \alpha \Delta T$$

6. Incompresisble Navier-Stokes-Boussinesq-Fourier

$$\nabla \cdot \mathbf{v} = 0$$
$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho_0} \nabla \left(p - \rho_0 g z \right) + \nu \Delta \mathbf{v} - \mathbf{g} B \left(T - T_0 \right)$$
$$\partial_t \left(\rho c_p T \right) + \nabla \cdot \left(\rho c_p T \mathbf{v} \right) = \nabla \cdot \left(\kappa \nabla T \right) + \mathbf{S}$$

Constitutive and closure relations

1. Newtonian fluid

$$\boldsymbol{\tau} = \lambda (\nabla \cdot \mathbf{v}) \mathbf{I} + \eta \left(\nabla \mathbf{v} + \nabla \mathbf{v}^T \right),$$

2. Hooke's law

$$\boldsymbol{\sigma} = \lambda \mathrm{tr}(\mathbf{D}) \mathbf{I} + 2 \mu \mathbf{D},$$

3. Boussinesq approximation

$$\rho = \rho_0 (1 - B(T - T_0))$$

4. Stefan condition

$$\rho_s L \partial_t X_m(t) = -\kappa \partial_x T(X_m^-(t), t) + \kappa \partial_x T(X_m^+(t), t)$$

Integration

1. $$\int exp(k\,x)\,dx = exp(k\,x) + C$$

2. $$\int x\,exp(k\,x)\,dx = exp(k\,x)(k\,x - 1)/k^2 + C$$