Model Based Estimation Methods: Exercise 12

Jaeyong Jung (359804), Sangwook Lee (362209)

July 20, 2016

Bonus problem 12

a.) Plot the estimated and true values of the heights in the two tanks (h_1, h_2).

True values are described as blue curves and estimated values as red curves on Figure 1. Deriving A, C, W, and V is done using Symbolic toolbox in MATLAB, see sdiff.m. Implementation is done on Kalman_filter_two_tank.m.

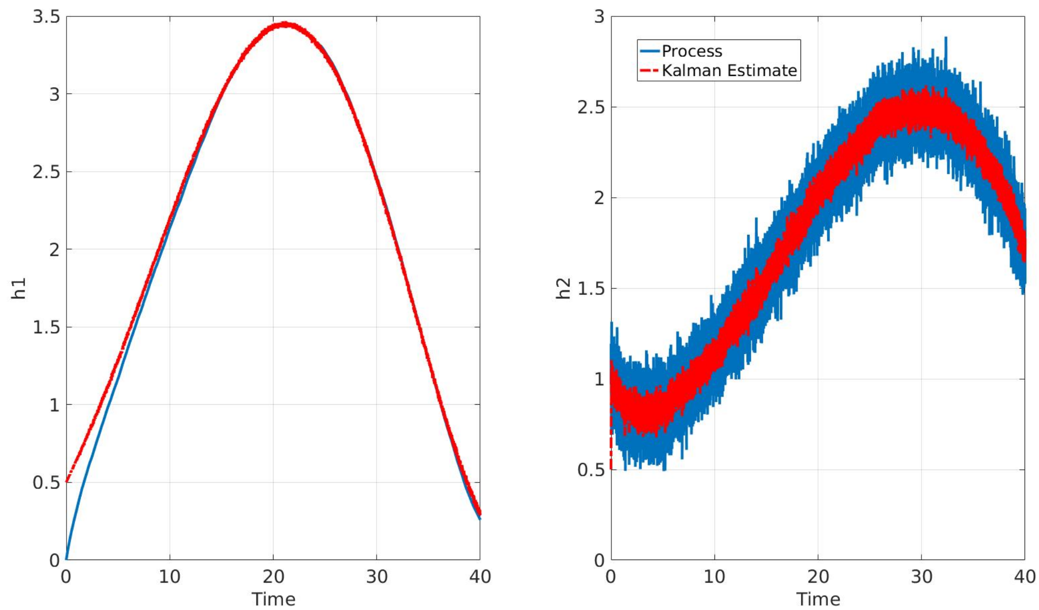

Figure 1: Comparing true and estimate values

b) Adjust the tuning parameters of the Kalman filter to get satisfactory results.

Initial parameters are described below.

$$\hat{x0} = [0.5, 0.5]'; P0 = [1e-2, 0; 0, 1e-2]; Q = [1e-1, 0; 0, 1e-1]; R = 1e-0;$$

$P0, Q, R$ can be tuned to improve quality of estimation. To investigate how those parameters influence the estimation, the effect is studied for each parameter.

Firstly, let's consider P0. If P0 is large, it means that $x\hat{(0)}$ is not considered to be good guess for $x(0)$. If P0 is set as $\text{diag}(10^2, 10^2)$, initial estimation of h_1 oscillates too much like Figure 2. Thus, it is not a good idea to increase P0. On the other hand, P0 is set as $\text{diag}(10^-5, 10^-5)$, but it does not make the graph different from original one.

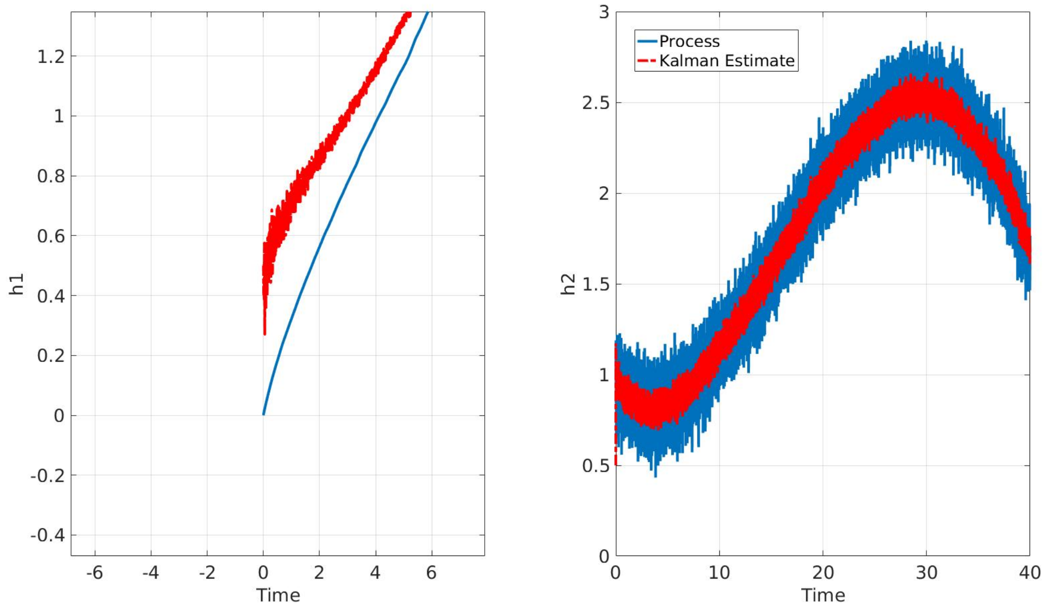

Figure 2: P0 is $\mathrm{diag}(10^2, 10^2)$

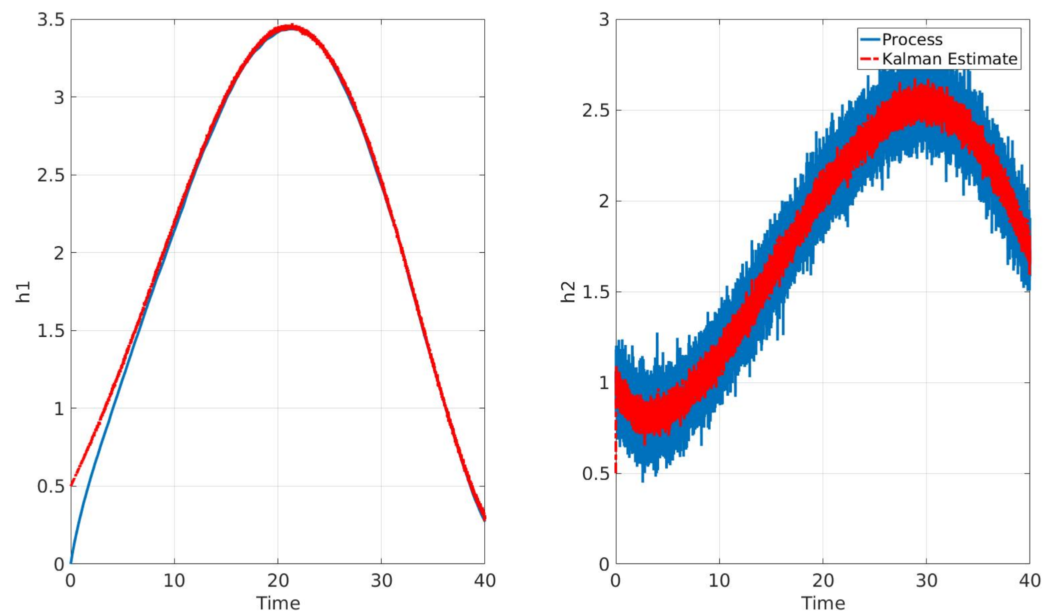

Figure 3: P0 is $\mathrm{diag}(10^-5, 10^-5)$

Next, state noise matrix Q is increased. to be $\mathrm{diag}(10^{+}4, 10^{+}4)$. It yields worse result as shown on Figure 4. Thus, Q is changed to be $\mathrm{diag}(10^{-}4, 10^{-}4)$ and examined.

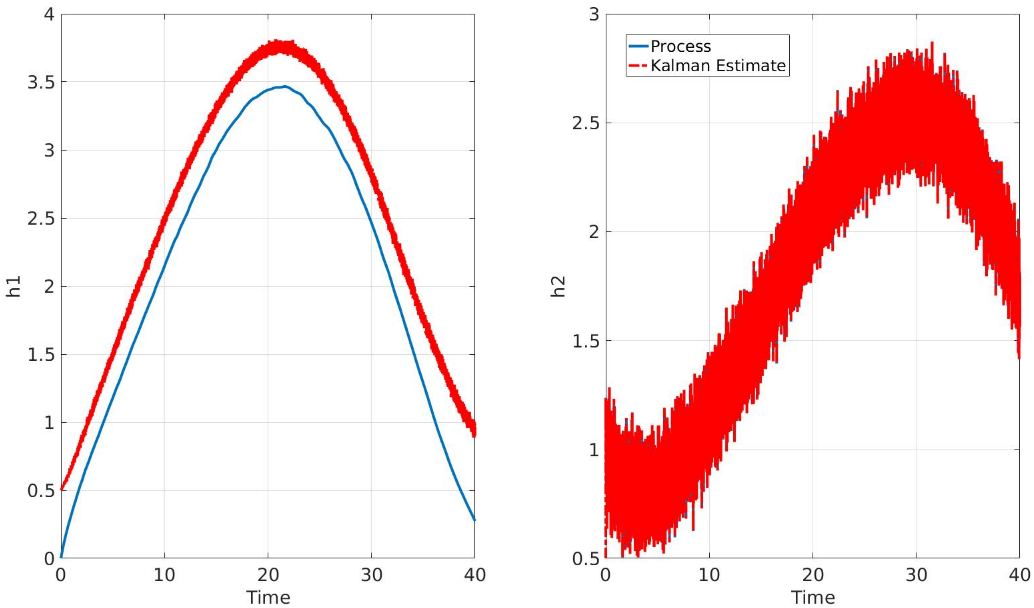

Figure 4: Q is $\mathrm{diag}(10^{+}4, 10^{+}4)$

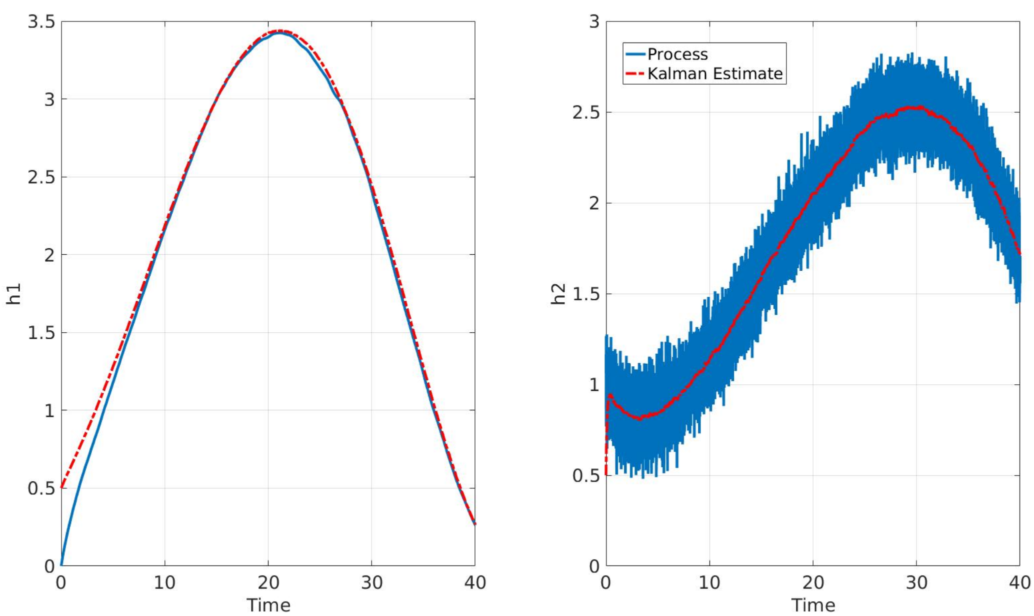

Figure 5: Q is $\mathrm{diag}(10^{-}4, 10^{-}4)$

On Figure 5, estimation is done in more clear way. It seems to have got rid of noise compared to Figure 1.

Finally, effect of R is studied. R is increased to 1000. It also makes the estimation more clear on Figure 6. However, decreasing R will make the estimation worse like Figure 7.

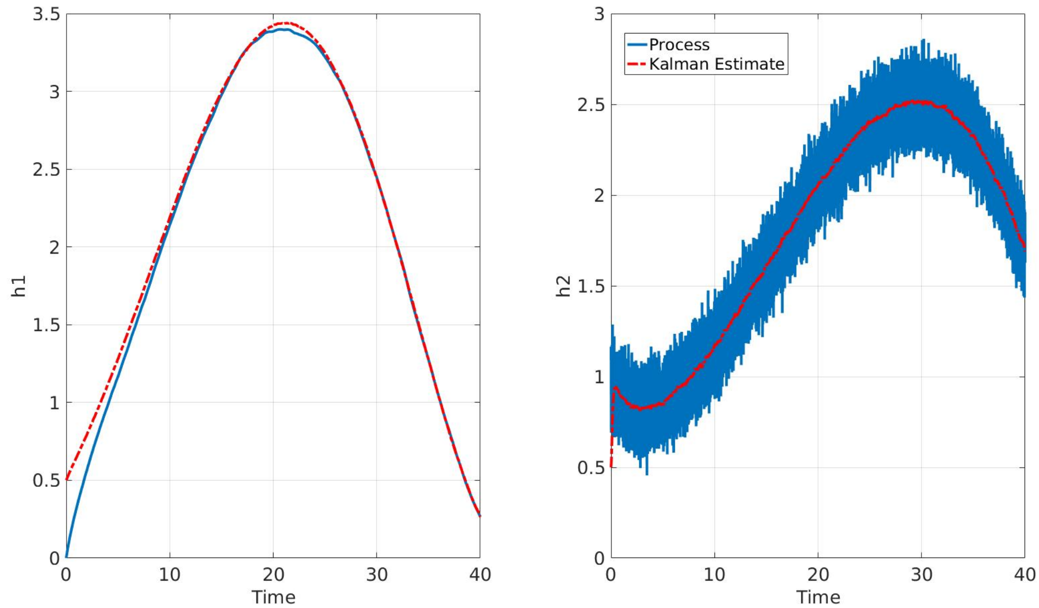

Figure 6: R is 1000

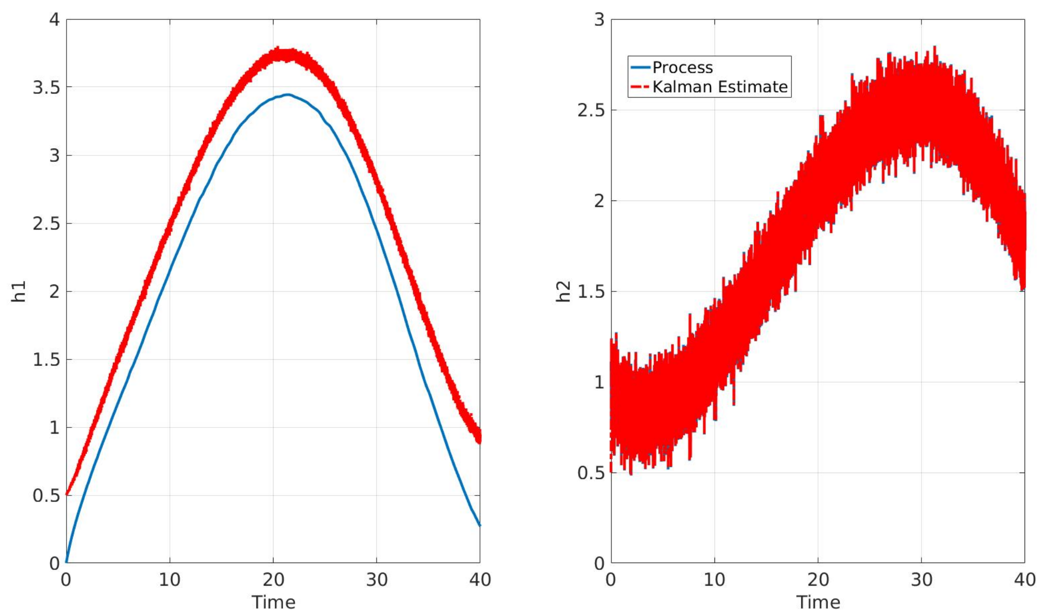

Figure 7: R is 0.001

In summary, decreasing Q and increasing R makes the estimation better. Optimal way will be mixing effects of Q and R. After several attempts, Q is set as diag$(10^{-}2, 10^{-}2)$ and R as 100. Figure 8 proves that this method works.

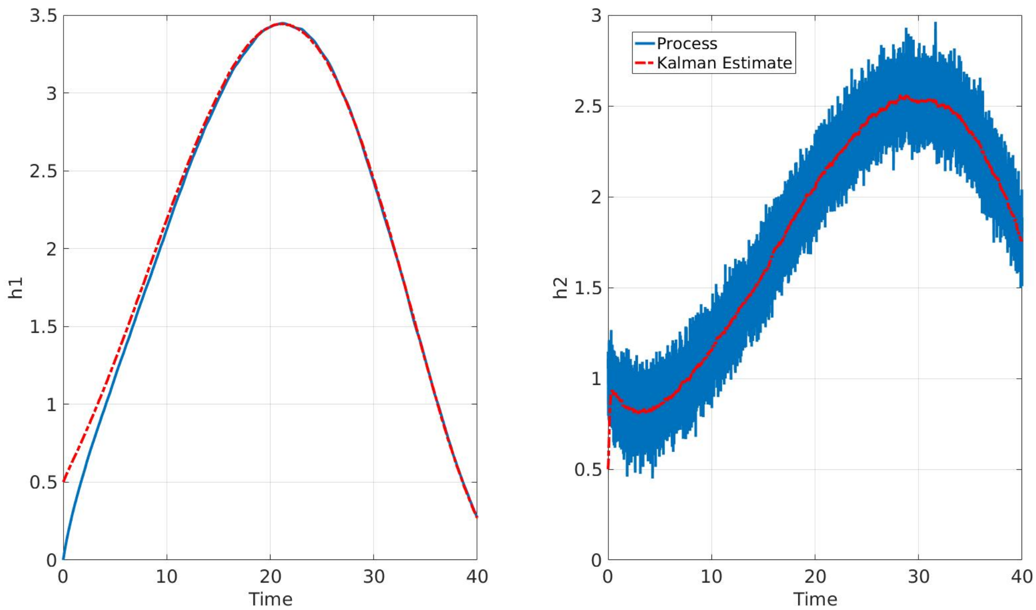

Figure 8: Q and R are diag$(10^{-}2, 10^{-}2)$ and 100

If initial guess of state variable can be modified, it will be the most critical factor. Initial guess $x_0 = [0.1, 1.1]$ improves Figure 8 and it has nice guess near initial point, Figure 9.

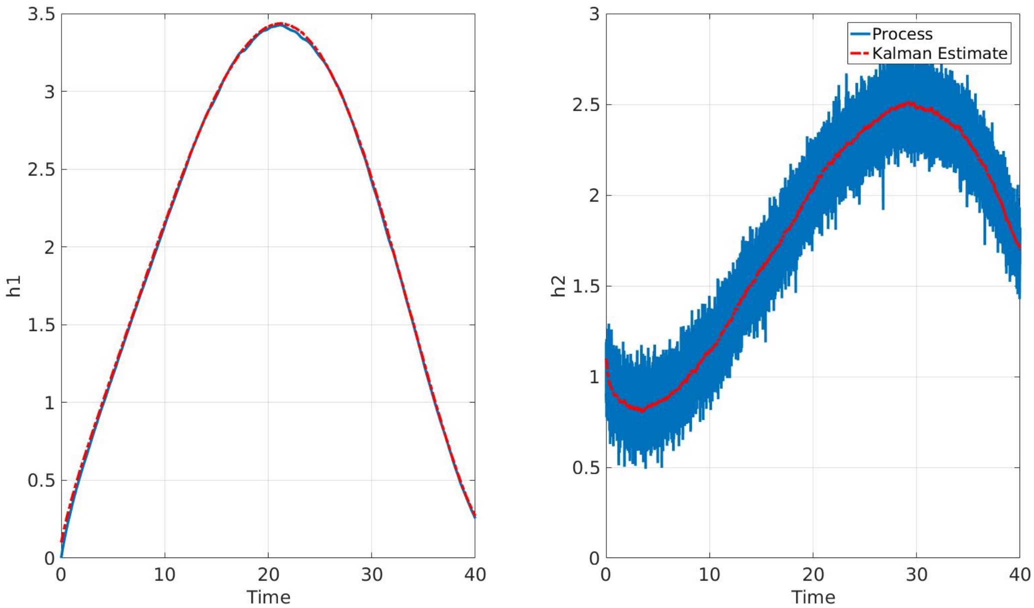

Figure 9: $x_0 = [0.1, 1.1]$