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Model-based Estimation Methods, SS 2016

Exercise 12 (July 13, 2016, 18:15 - 19:45)

Nonlinear State Estimation, Input Estimation

Problem 17: Input estimation via state-space extension (Matlab)

1. A motorcycle is traveling on a straight road. The velocity of the motorcycle can be manipulated by changing the acceleration. Measurements of the current location of the motorcycle are the only measurements that are available. Model the system to determine the position and velocity of the motorcycle at any time, assuming you have information about the acceleration of the motorcycle.

2. Discretize the system using the Euler method and write the model as a discrete time LTI system. Write down the A_d, B_d, C_d and D_d matrices.

3. Simulate this system for 40s, assuming a sampling time of 0.1s. Generate a set of noise-free and noisy measurements. Assume that the standard deviation of the noise is 2.5m.

   Use the following piece of code for the input profile:

```
tStart = 0;
deltat = 0.1;
tEnd = 40;
t = [tStart: deltat: tEnd]';
u = zeros(1,length(t));
u(round(length(t)/20):round(length(t)/10)) = 5;
u(round(length(t)/5):round(length(t)/5)+round(length(t)/20)) = 10;
u(round(length(t)*3/10):round(length(t)*3/10)+round(length(t)*3/40))
= -15;
u(round(length(t)*12/20):round(length(t)*12/20)+round(length(t)*2/10))
= 5;
```

4. Assume that the inputs are unknown and that you would like to estimate them using the Kalman Filter. Formulate the problem as an input estimation problem

with state vector extension. Write down the A_{ext} and C_{ext} matrices of the extended system. Is the extended system observable?

5. Implement a kalman filter to estimate the states and inputs of the extended discrete time system.

You may edit the code `Input_estimation_state_vector_extension.m`.

Bonus Problem 12: Extended Kalman Filter (Points(2)) - Matlab

(Upload your code to the L2P classroom before 20 July, 18:15)

A two-tank system is assumed to be modeled using a set of nonlinear differential equations given by:

$$\begin{aligned} A_1 \frac{dh_1}{dt}(t) &= F_{in}(t) - F_1(t) \\ A_2 \frac{dh_2}{dt}(t) &= F_1(t) - F_2(t), \end{aligned}$$

where

$$\begin{aligned} F_1(t) &= \sqrt{h_1(t)} R_1 \\ F_2(2) &= \sqrt{h_2(t)} R_2 \end{aligned}$$

R_1 and R_2 are resistances to flow. The parameters, initial conditions and inputs for the process are

$$\begin{aligned} R_1 &= R_2 = 1, \text{ resistances to the flows} \\ A_1 &= 2\ [m^2], \text{ cross sectional area of tank 1} \\ A_2 &= 5\ [m^2], \text{ cross sectional area of tank 2} \\ w_x &\sim N(0, \sigma_x^2), \sigma_x = [0.01, 0.01]' \text{ process noise} \\ v_x &\sim N(0, \sigma_y^2), \sigma_y = 0.1, \text{ measurement noise} \\ h_1(0) &= 0\ [m], \text{ initial height in tank 1} \\ h_2(0) &= 1\ [m], \text{ initial height in tank 2} \\ F_{in}(t) &= \sin(0.1t) + 1\ [m^3/s], \text{ input signal} \\ T &= 0.001\ [s], \text{ sample time} \\ t_{end} &= 40\ [s], \text{ simulation time} \end{aligned}$$

Assume that only h_2 can be measured.

(1) Implement an EKF for this system. Use the following parameters for the EKF implementation:

$$\begin{aligned}
\hat{x}^+[0] &= \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}' \\
P^+[0] &= \mathrm{diag}(0.01,\, 0.01) \\
Q &= \mathrm{diag}(0.1,\, 0.1) \\
R &= 1
\end{aligned}$$

Plot the estimated and true values of the heights in the two tanks (h_1, h_2).

(2) Adjust the tuning parameters of the Kalman filter to get satisfactory results.

Hint: In this case the linearized system matrices are time-invariant. For simplicity, you may calculate them by hand, instead of using the symbolic toolbox of MATLAB.