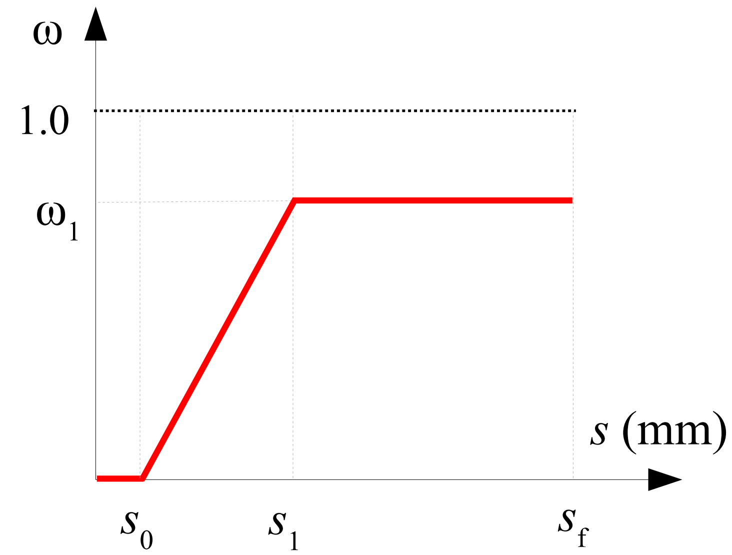
Derive a bond slip law from a given damage function

A tri-linear linear damage function shown in the figure is given as follows.

$$\omega(s) = \begin{cases} 0 & s \leq s_0 \\ \omega_1 \left[1 - \dfrac{s_1 - s}{s_1 - s_0} \right] & s_0 < s \leq s_1 \\ \omega_1 & s_1 < s \leq s_f \end{cases}$$



Assuming that a bond behavior is governed purely by damage and provided the characteristic points of the damage function are given as

$\omega_1 = 0.8$, $s_0 = 0.1$ mm, $s_1 = 0.4$ mm, $s_f = 1.0$ mm, $Eb = 100$ MPa/mm

a) Derive the bond-slip law governed by the given damage function
b) Calculate the bond stress at the slip $s = 0.25$ mm.
c) Calculate the unloading stiffness at the slip $s = 0.25$ mm.
d) Sketch graphically the derived bond-slip law.

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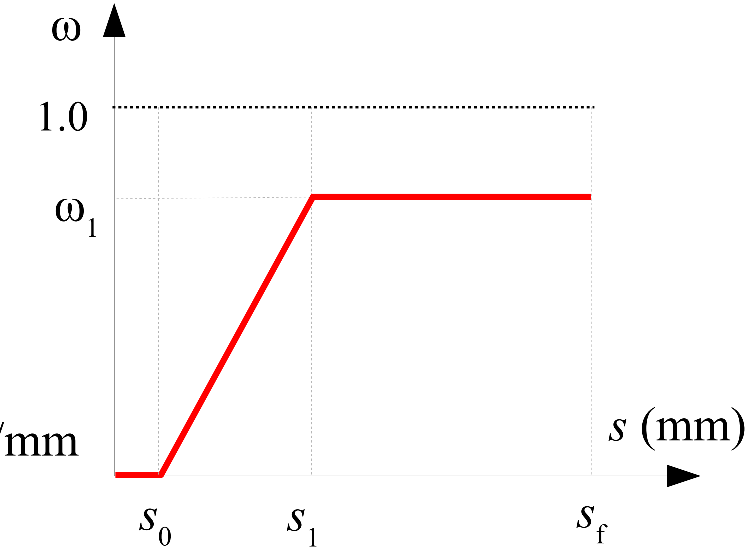
$$\omega(s) = \begin{cases} 0 & s \le s_0 \\ \omega_1 \left[1 - \dfrac{s_1 - s}{s_1 - s_0} \right] & s_0 < s \le s_1 \\ \omega_1 & s_1 < s \le s_\mathrm{f} \end{cases}$$



$\omega_1 = 0.8$, $s_0 = 0.1$ mm, $s_1 = 0.4$ mm, $s_\mathrm{f} = 1.0$ mm, $Eb = 100$ MPa/mm

a) Derive the bond-slip law governed by the given damage function.

Solution:

$$\tau(s) = [1 - \omega(s)]\; E_\mathrm{b}\; s$$

e.g. branch 1 ($s \le s_0$) : $\tau(s) = [1 - 0]\; E_\mathrm{b}\; s \longrightarrow \tau(s) = E_\mathrm{b}\; s$

$$\tau(s) = \begin{cases} E_\mathrm{b}\; s & s \le s_0 \\ \left[1 - \omega_1 \left(1 - \dfrac{s_1 - s}{s_1 - s_0} \right) \right] E_\mathrm{b}\; s & s_0 < s \le s_1 \\ (1 - \omega_1) E_\mathrm{b}\; s & s_1 < s \le s_\mathrm{f} \end{cases}$$

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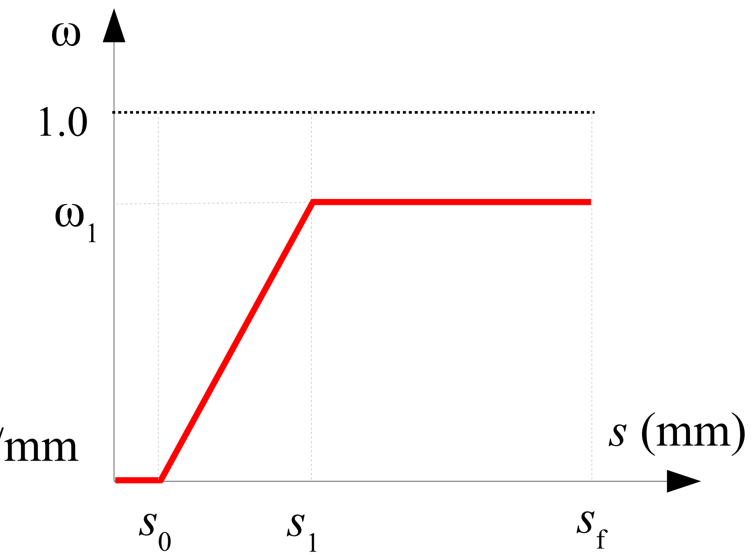
$$\omega(s) = \begin{cases} 0 & s \leq s_0 \\ \omega_1 \left[1 - \dfrac{s_1 - s}{s_1 - s_0}\right] & s_0 < s \leq s_1 \\ \omega_1 & s_1 < s \leq s_\mathrm{f} \end{cases}$$



$\omega_1 = 0.8$, $s_0 = 0.1$ mm, $s_1 = 0.4$ mm, $s_f = 1.0$ mm, $Eb = 100$ MPa/mm

b) Calculate the bond stress at the slip $s = 0.25$ mm.

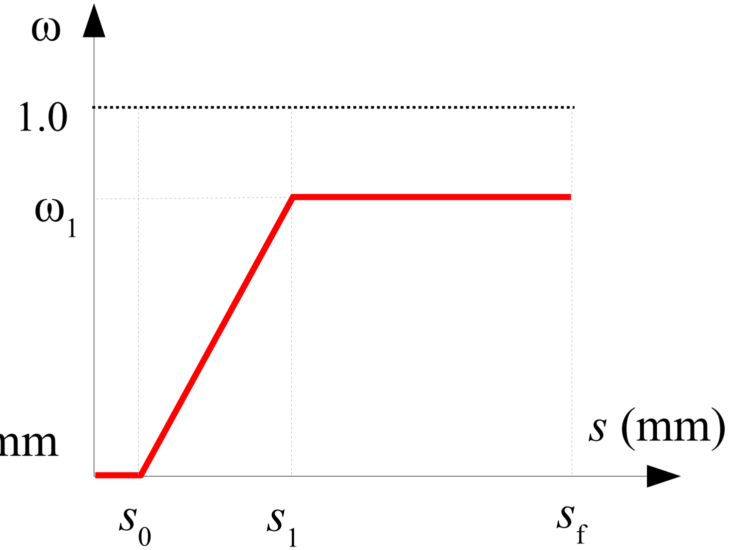
Solution:

$$\tau(s) = \begin{cases} E_\mathrm{b}\ s & s \leq s_0 \\ \left[1 - \omega_1 \left(1 - \dfrac{s_1 - s}{s_1 - s_0}\right)\right] E_\mathrm{b}\ s & s_0 < s \leq s_1 \\ (1 - \omega_1) E_\mathrm{b}\ s & s_1 < s \leq s_\mathrm{f} \end{cases}$$

$$\tau(s = 0.25) = \left[1 - 0.8 \left(1 - \frac{0.4 - 0.25}{0.4 - 0.1}\right)\right] 100 \times 0.25 = 15\mathrm{MPa}$$

Derive a bond slip law from a given damage function

$$\omega(s) = \begin{cases} 0 & s \leq s_0 \\ \omega_1 \left[1 - \dfrac{s_1 - s}{s_1 - s_0}\right] & s_0 < s \leq s_1 \\ \omega_1 & s_1 < s \leq s_{\mathrm{f}} \end{cases}$$



$\omega_1 = 0.8$, $s_0 = 0.1$ mm, $s_1 = 0.4$ mm, $s_f = 1.0$ mm, $Eb = 100$ MPa/mm

c) Calculate the unloading stiffness at the slip $s = 0.25$ mm.

Solution:

$$E(\omega) = (1 - \omega)E_{\mathrm{b}}$$

$$\omega(s) = \begin{cases} 0 & s \leq s_0 \\ \omega_1 \left[1 - \dfrac{s_1 - s}{s_1 - s_0}\right] & s_0 < s \leq s_1 \\ \omega_1 & s_1 < s \leq s_{\mathrm{f}} \end{cases} \Rightarrow \omega(s = 0.25) = 0.8 \left[1 - \dfrac{0.4 - 0.25}{0.4 - 0.1}\right] = 0.4$$

$$\Rightarrow E(\omega = 0.4) = (1 - 0.4)100 = 60 \ [\mathrm{MPa/mm}]$$

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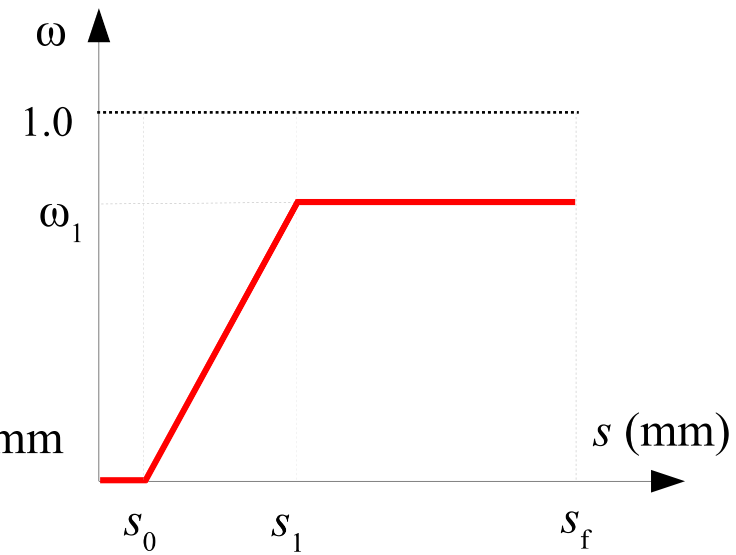
$$\omega(s) = \begin{cases} 0 & s \leq s_0 \\ \omega_1 \left[1 - \dfrac{s_1 - s}{s_1 - s_0}\right] & s_0 < s \leq s_1 \\ \omega_1 & s_1 < s \leq s_\mathrm{f} \end{cases}$$



$\omega_1 = 0.8$, $s_0 = 0.1$ mm, $s_1 = 0.4$ mm, $s_\mathrm{f} = 1.0$ mm, $Eb = 100$ MPa/mm

d) Sketch graphically the derived bond-slip law.

Solution:

$\tau(s = 0.1) = 10$ MPa (elastic range)

$\tau(s = 0.25) = 15$ MPa (from task(b))

For the slip s=0.4 and s=1.0 (third branch)

$$\tau(s) = (1 - \omega_1) E_\mathrm{b}\ s$$

$$\tau(s = 0.4) = (1 - 0.8) 100 \times 0.4 = 8.0\,\mathrm{MPa}$$

$$\tau(s = 1.0) = (1 - 0.8) 100 \times 1.0 = 20.0\ \mathrm{MPa}$$