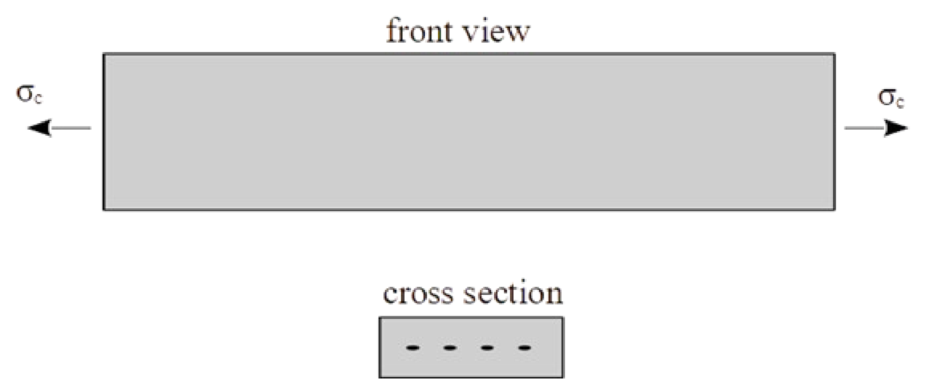
X0204: Tensile behavior of a composite with constant bond-slip law

For the displayed tensile test of textile-reinforced composite specimen assuming a constant bod-slip law with the given data:



Fiber: fiber strength $\sigma_{fu} = 3500$ [Mpa]
$E_f = 240000$ [MPa]
reinforcement ratio = 1.0 %
perimeter p = 50 [mm]

Matrix: matrix strength $\sigma_{mu} = 3.0$ [MPa]
$E_m = 30000$ [MPa], $A_m = 1500$ [mm2]

Bond: $\tau = 5$ [MPa]

a) Calculate the stiffness of the composite.

b) Calculate the characteristic values of the ACK model and plot the stress-strain response of the composite.

c) Calculate the average crack spacing.

d) If the reinforcement ratio is doubled, how will the stress-strain response and the average crack spacing change? (Assume p = 100 [mm] and $A_m = 1500$ [mm^2]).

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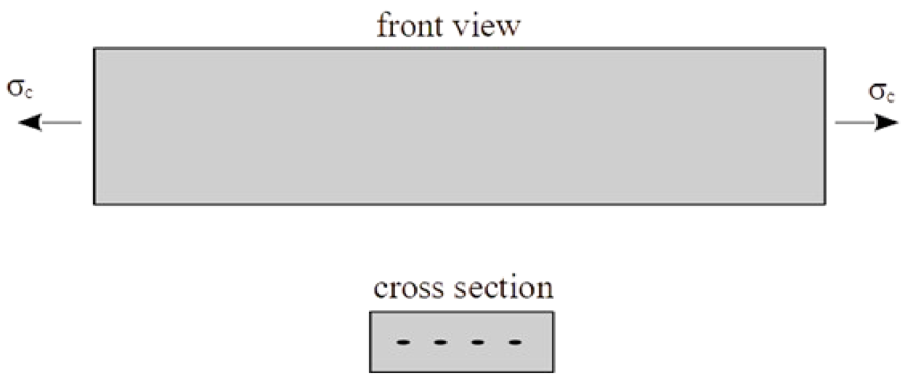

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Matrix:	matrix strength σ_{mu} = 3.0 [MPa] E_m = 30000 [MPa], A_m = 1500 [mm2]
Bond:	τ = 5 [MPa]

a) Calculate the stiffness of the composite.

Solution:

Composite stiffness:

$$E_c = E_m \text{ x } (1 - V_f) + E_f \text{ x } V_f$$

$$E_c = 30000 \text{ x } (1 - 0.01) + 24000 \text{ x } 0.1 = 29940 \text{ [MPa]}$$

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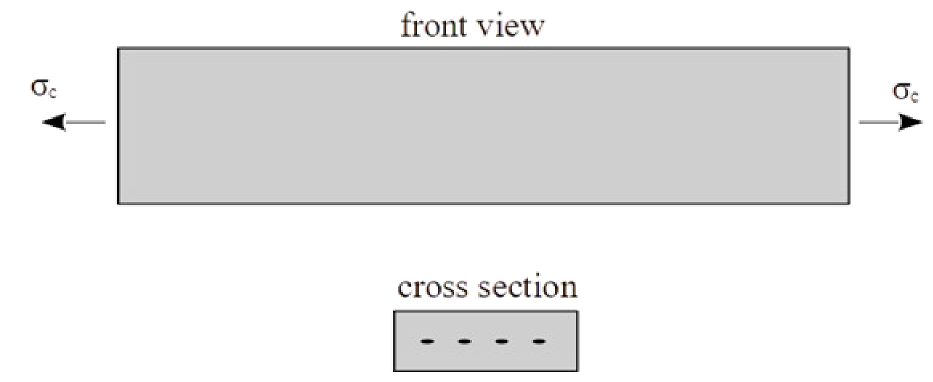

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Bond: $\tau = 5$ [MPa]

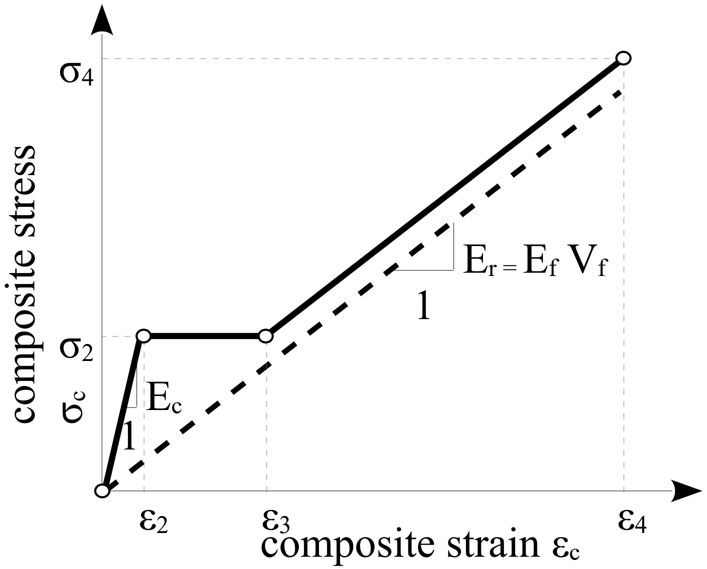
b) Calculate the characteristic values of the ACK model and plot the stress-strain response of the composite.

Solution:

Point 1: $(\varepsilon_1, \sigma_1) \rightarrow (0\ ,\ 0)$

Point 2: $(\varepsilon_2, \sigma_2) \rightarrow \varepsilon_2 = \sigma_{mu} / E_m = 3 / 30000 = 0.0001$

$$\sigma_2 = E_c \times \varepsilon_2 = 29940 \times 0.0001 = 2.994 \text{ [MPa]}$$



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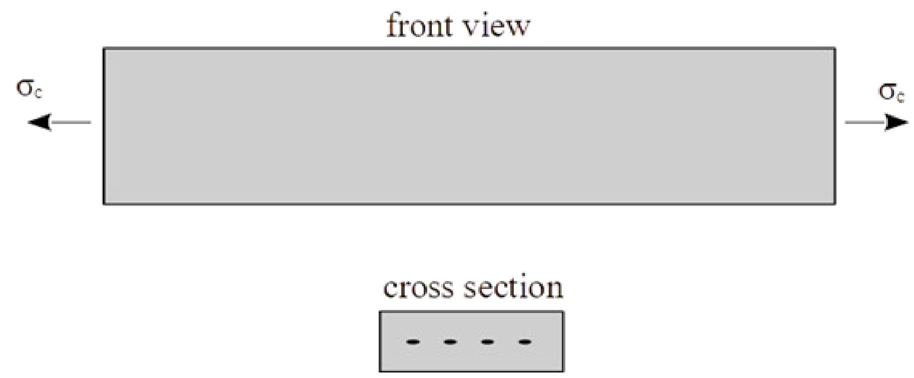

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$E_m = 30000$ [MPa], $A_m = 1500$ [mm2]

Bond: $\tau = 5$ [MPa]

b) Calculate the characteristic values of the ACK model and plot the stress-strain response of the composite.

Solution: Point 3: (ε_3, σ_3):

$\varepsilon_3 = (\sigma_{mu} / E_m) \text{ x } (1 + 0.666 \text{ x } \alpha_e)$

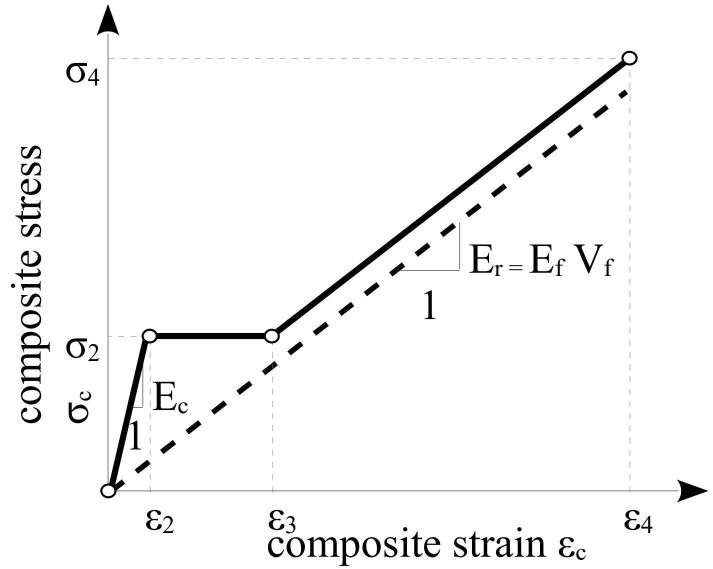
$\alpha_e = [E_m \text{ x } (1 - V_f)] / [E_f \text{ x } V_f] = [30000 \text{ x } (1 - 0.01)] / [240000 \text{ x } 0.01]$

$= 12.375$

$\varepsilon_3 = (\sigma_{mu} / E_m) \text{ x } (1 + 0.666 \text{ x } \alpha_e) = (3 / 30000) \text{ x } (1 + 0.666 \text{ x } 12.375)$

$= 0.000924$

$\sigma_3 = \sigma_2 = 2.994$ [MPa]



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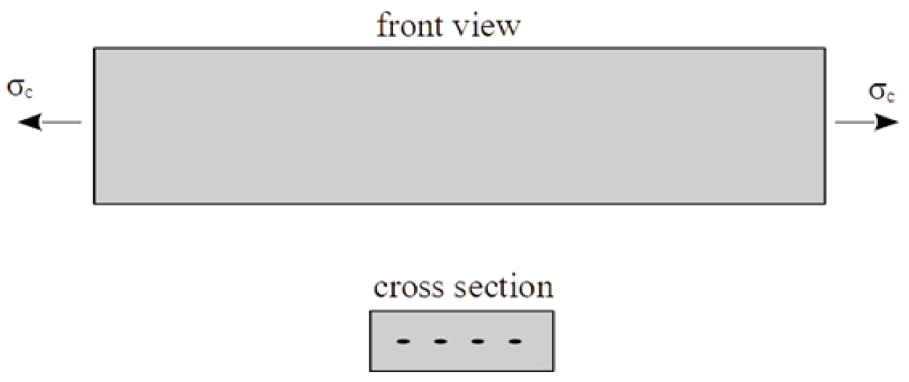

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$E_m = 30000$ [MPa], $A_m = 1500$ [mm2]

Bond: $\tau = 5$ [MPa]

b) Calculate the characteristic values of the ACK model and plot the stress-strain response of the composite.

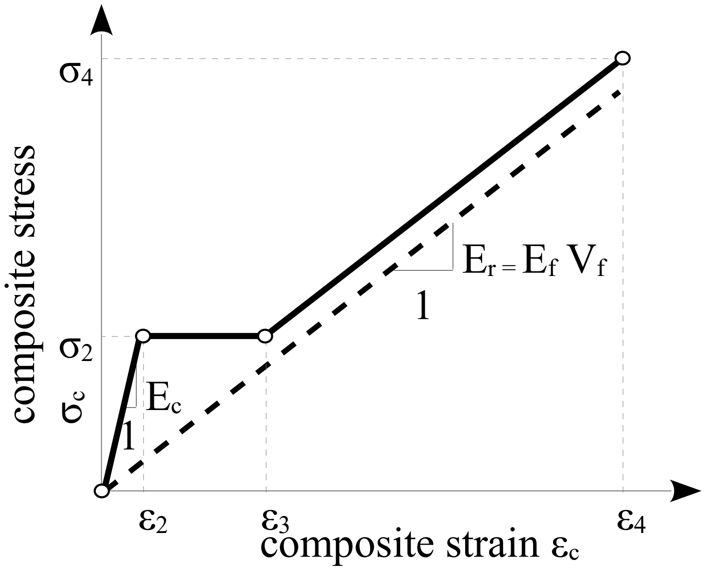
Solution: Point 4: (ε_4, σ_4):

$$\varepsilon_4 = \varepsilon_3 + (\sigma_4 - \sigma_2) / E_r$$

$$E_r = E_f \times V_f = 240000 \times 0.01 = 2400 \text{ [Mpa]}$$

$$\sigma_4 = \sigma_{fu} \times V_f = 3500 \times 0.01 = 35 \text{ [Mpa]}$$

$$\varepsilon_4 = 0.000924 + (35 - 2.994) / 2400 = 0.0143$$



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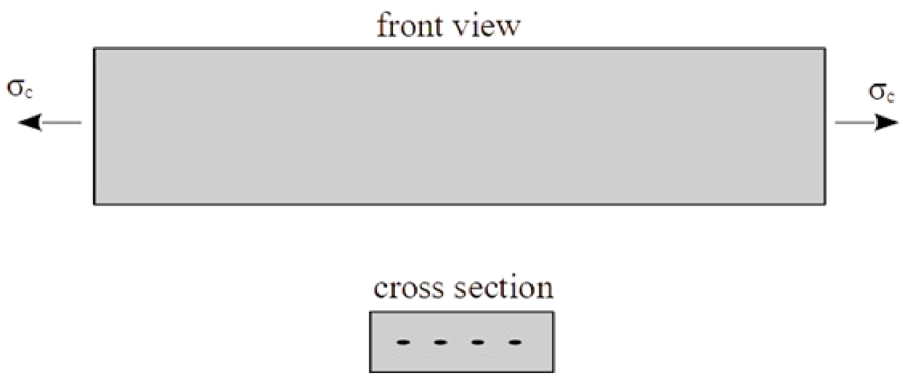

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Matrix:	matrix strength σ_{mu} = 3.0 [MPa] E_m = 30000 [MPa], A_m = 1500 [mm2]
Bond:	τ = 5 [MPa]

c) Calculate the average crack spacing.

Solution:

$$l_{cs} = 1.337 \text{ x } l_{shielded} = 1.337 \text{ x } (A_m \text{ x } \sigma_{mu}) / (\tau \text{ x } p)$$

$$= 1.337 \text{ x } (1500 \text{ x } 3) / (5 \text{ x } 50) = 24.066 \text{ [mm]}$$

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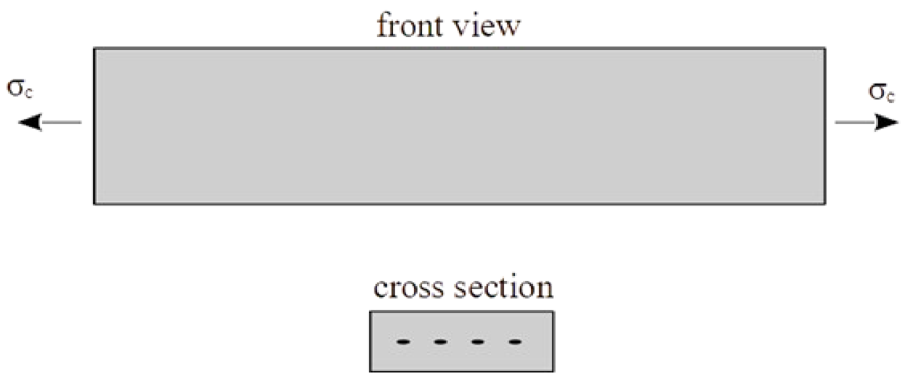

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Bond: τ = 5 [MPa]

d) If the reinforcement ratio is doubled, how will the stress-strain response and the average crack spacing change? (Assume p = 100 [mm] and A_m = 1500 [mm^2])..

Solution: similar to the tasks (a-c)

Composite stiffness: E_c = 34200 [MPa]

ACK: Point 1: $(\varepsilon_1, \sigma_1) \rightarrow (0 , 0)$
Point 2: $(\varepsilon_2, \sigma_2) \rightarrow (0.00010 , 3.42)$
Point 3: $(\varepsilon_3, \sigma_3) \rightarrow (0.00051 , 3.42)$
Point 4: $(\varepsilon_4, \sigma_4) \rightarrow (0.01438 , 70)$

l_{cs} = 1.337 x (1500 x 3) / (5 x 100) = 12.033 [mm]