bmcs course

classification of pull-out configurations using constant bond-slip laws

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Pullout with constant bond slip law – small but general playground

Develop sound understanding of the mechanisms in the pull-out response

$$\tau(x), \varepsilon(x), \sigma(x), u(x)$$
$$P(w)$$

Is it possible to systematically derive analytical solutions for changed boundary conditions?

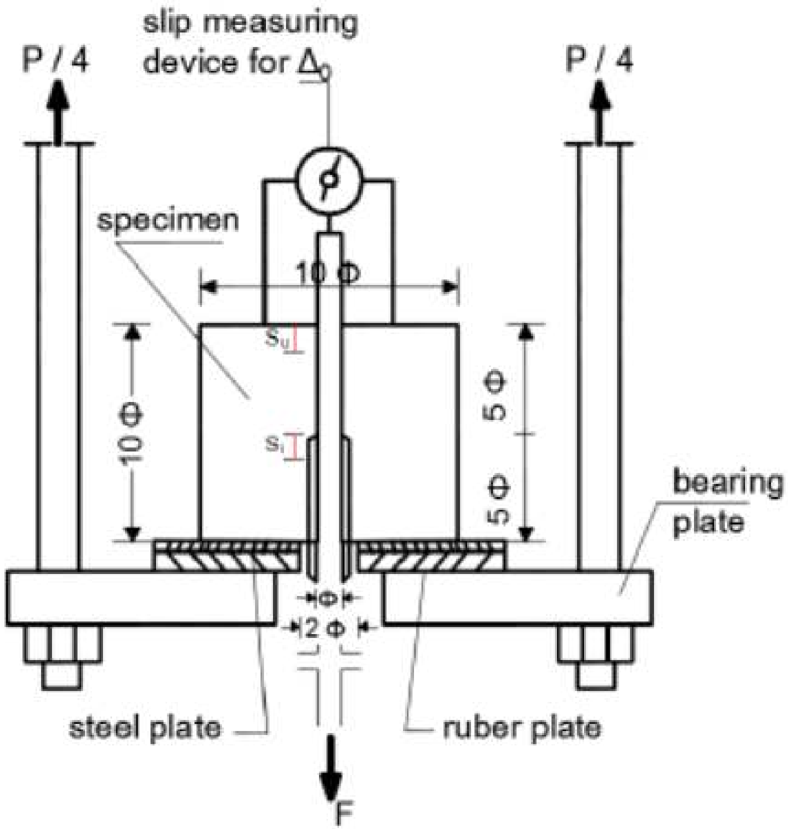
Effect of modified boundary conditions?

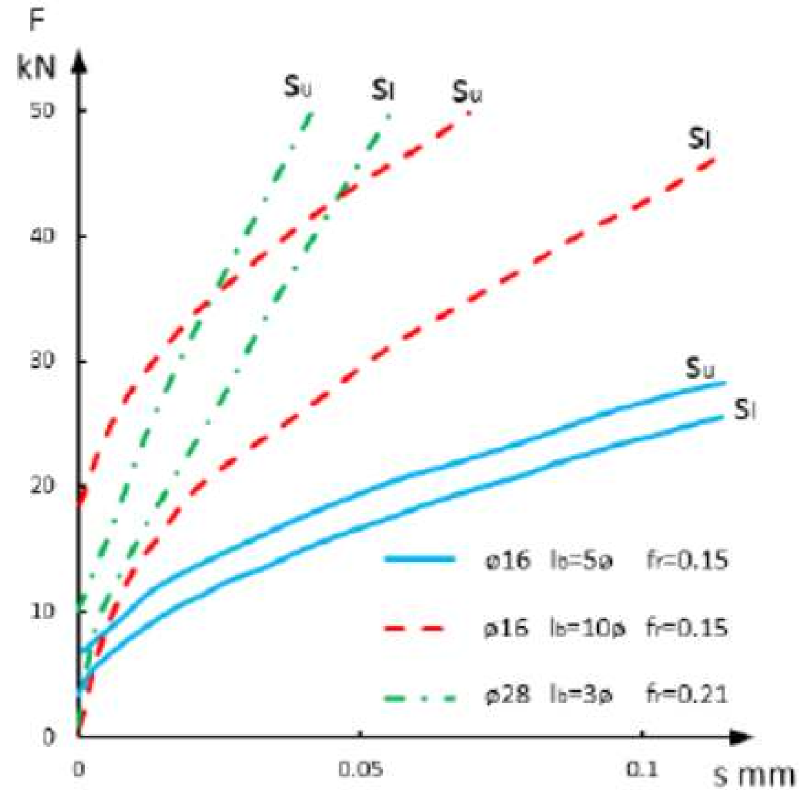
Effect of model parameters?

$$\bar{\tau}, p, A_\mathrm{f}, E_\mathrm{f}, A_\mathrm{m}, E_\mathrm{m}, L_\mathrm{b}$$

Validity of the model – what kind of pull-out test can be predicted and what can not?

Considered configurations of a test setup





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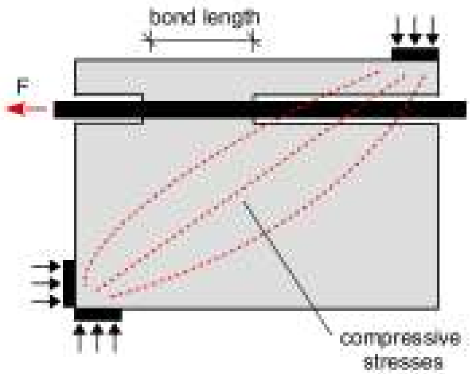
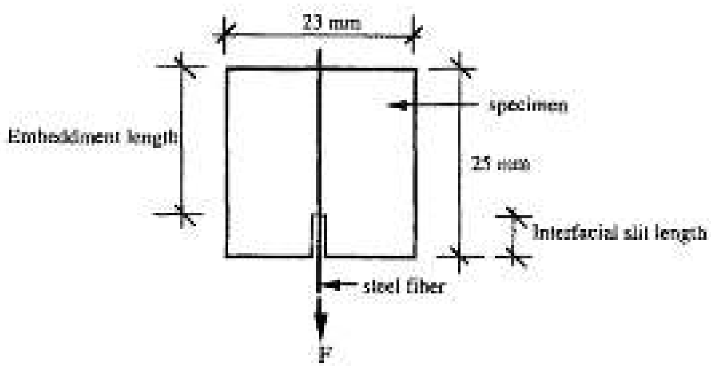
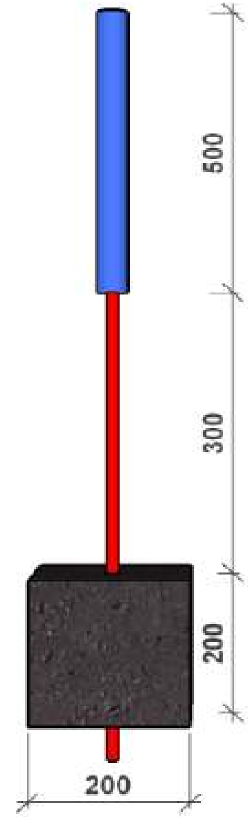
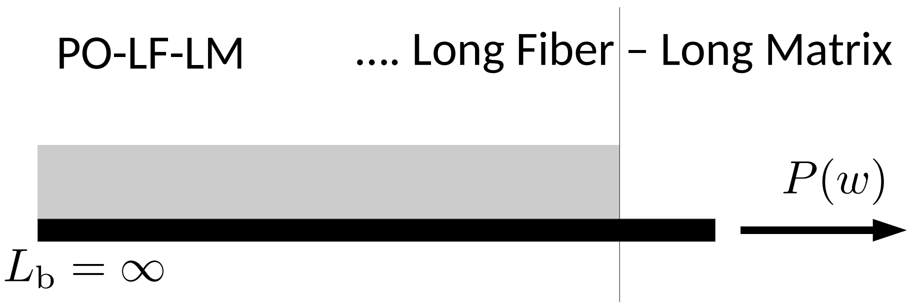






Pull-out test variants

PO-LF-LM .... Long Fiber – Long Matrix



$$P(w) = \sqrt{2p\bar{\tau}E_\mathrm{f}A_\mathrm{f}w}$$

Pull-out test variants

Material behavior **R - rigid** $E_\mathrm{m} A_\mathrm{m} = \infty$ **E - elastic** $E_\mathrm{m} A_\mathrm{m} \in (0, \infty)$

PO-ELF-RLM .... Elastic Long Fiber – Rigid Long Matrix

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PO-ELF-RLM

PO-ELF-RSM …. Elastic Long Fiber – Rigid Short Matrix

Pull-out test variants

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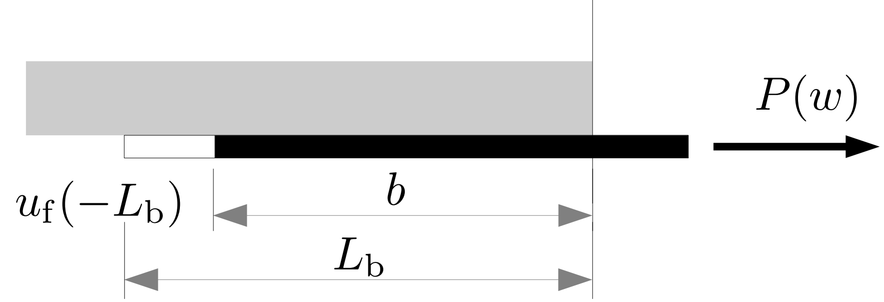
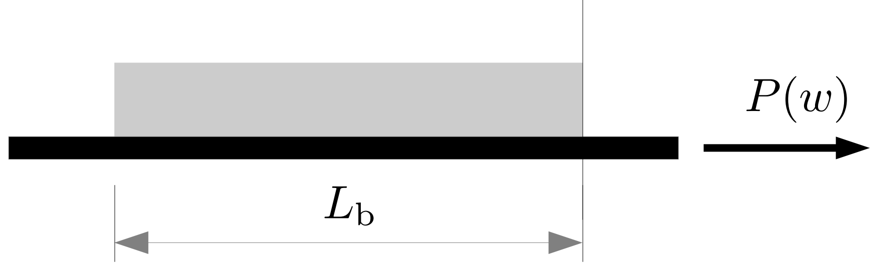
PO-ELF-RLM

PO-ESF-RLM .... Elastic Short Fiber



PO-ELF-RSM



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PO-ELF-RLM

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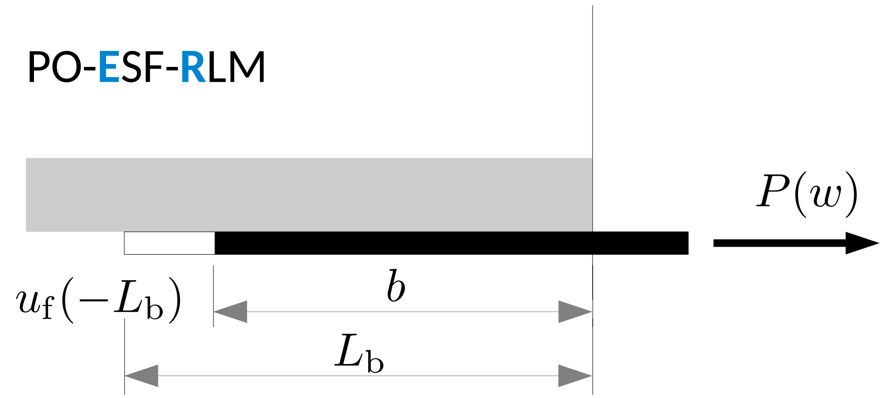
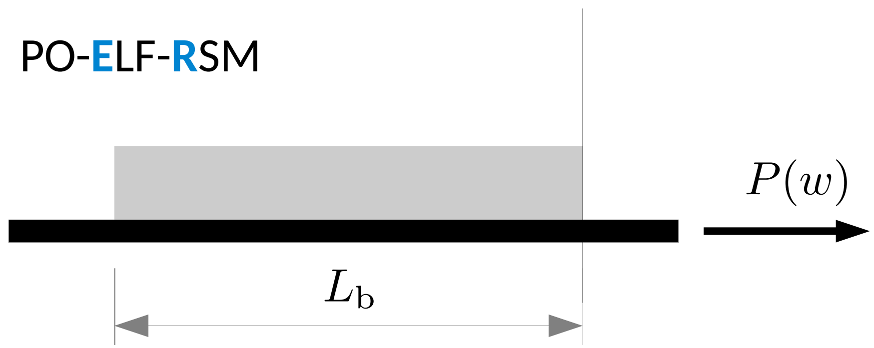
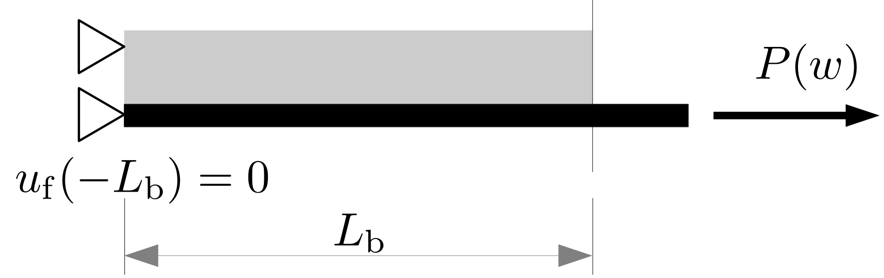


PO-ELF-RSM

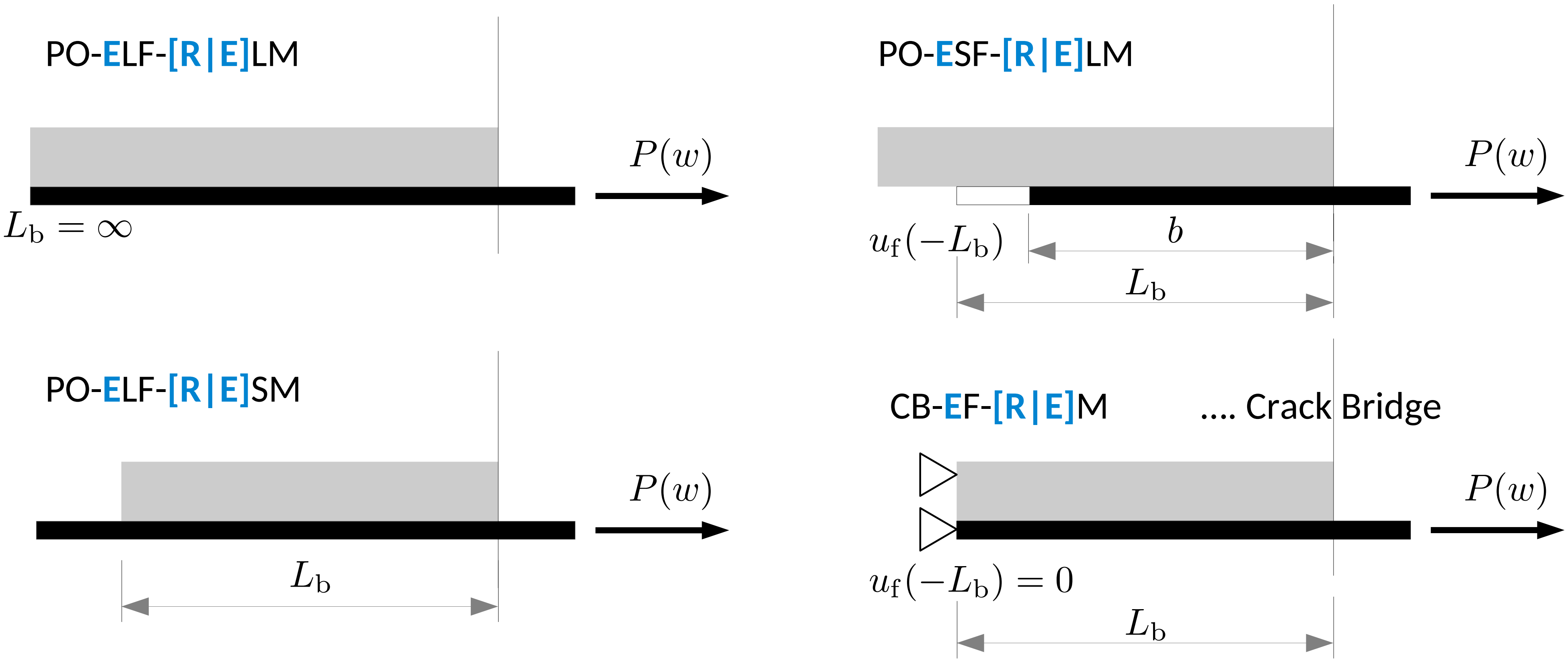


CB-EF-RM .... Crack Bridge



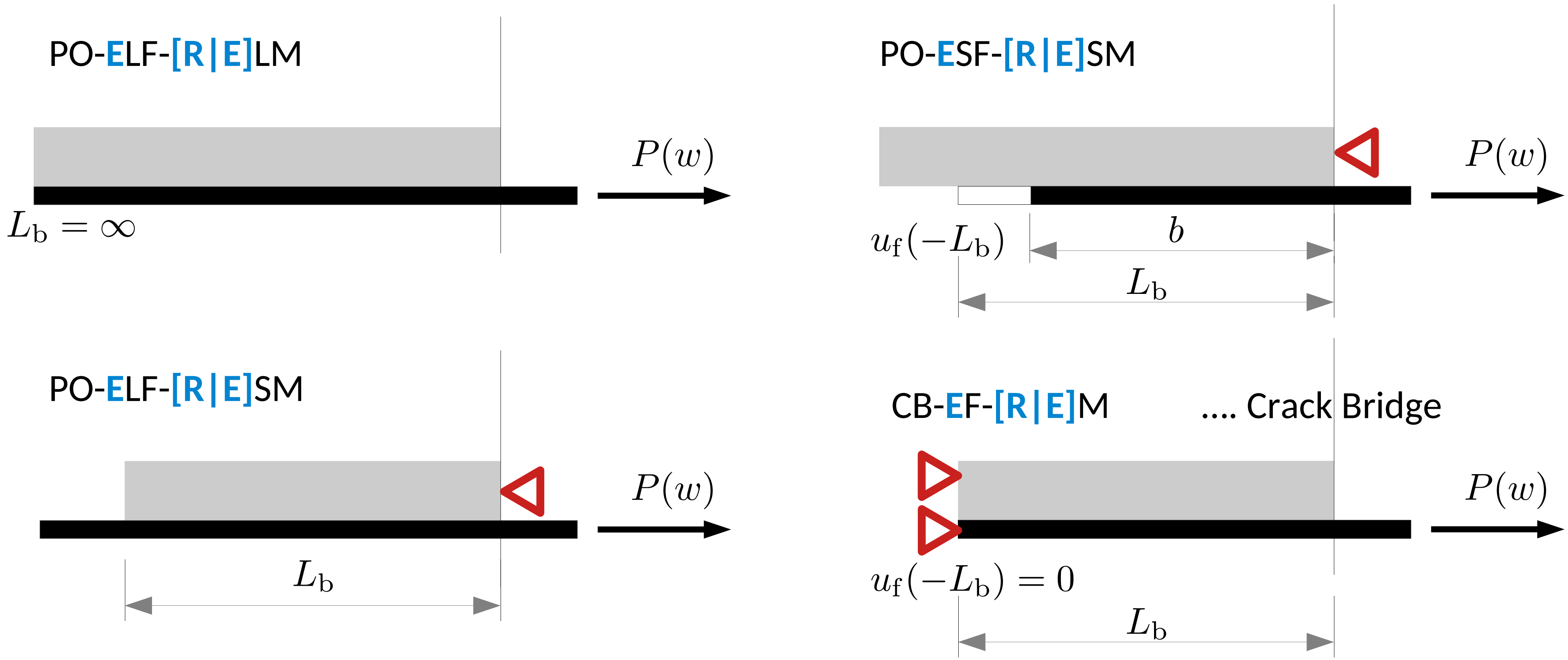
Pull-out test variants – Elastic Matrix

Material behavior **R - rigid** $E_\mathrm{m} A_\mathrm{m} = \infty$ **E - elastic** $E_\mathrm{m} A_\mathrm{m} \in (0, \infty)$



Pull-out test variants – Elastic Matrix – Support Position

Material behavior **R - rigid** $E_\mathrm{m} A_\mathrm{m} = \infty$ **E - elastic** $E_\mathrm{m} A_\mathrm{m} \in (0, \infty)$



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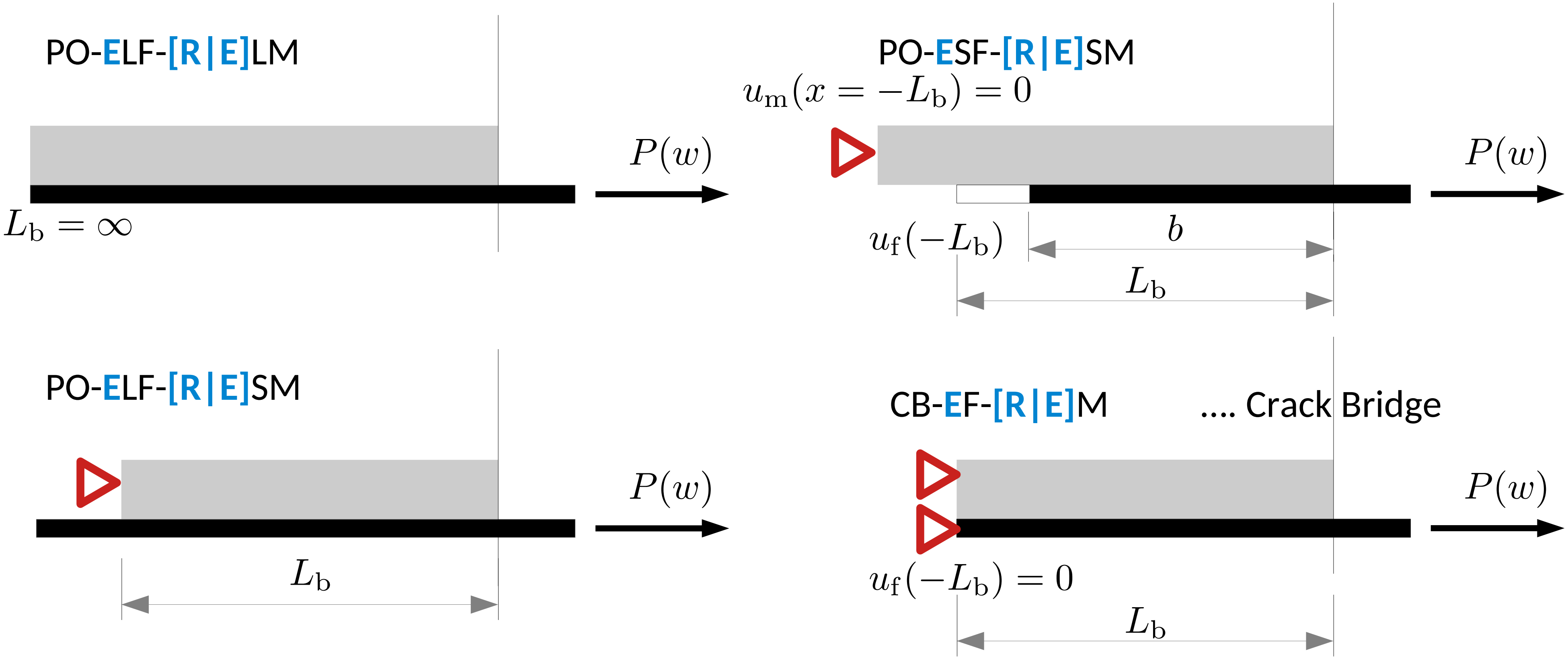


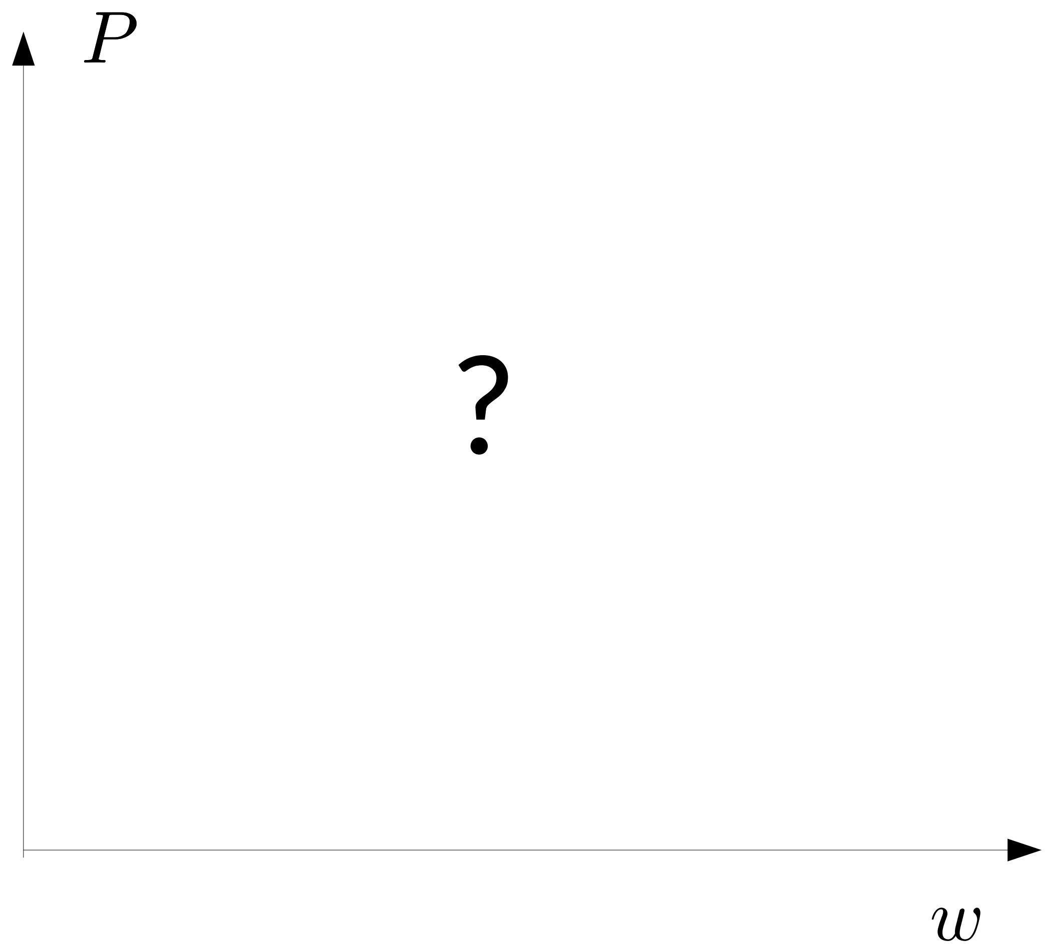
Qualitative comparison of the selected pullout configurations

2.1 - PO-ELF-R[L|S]M

2.2 - PO-ELF-ELM

2.3 - PO-ESF-RLM

2.4 - CB-EF-RM



2.1 PO-ELF-RLM

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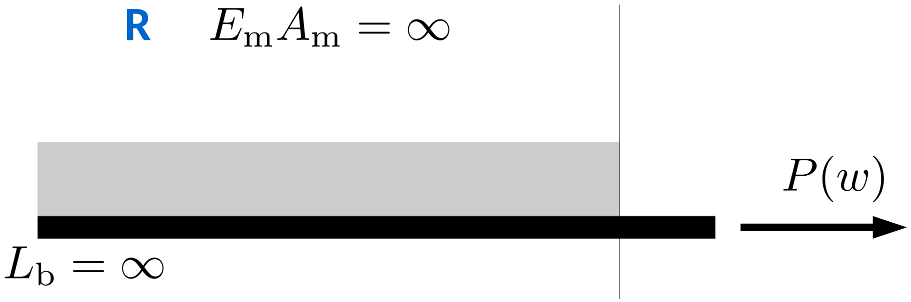
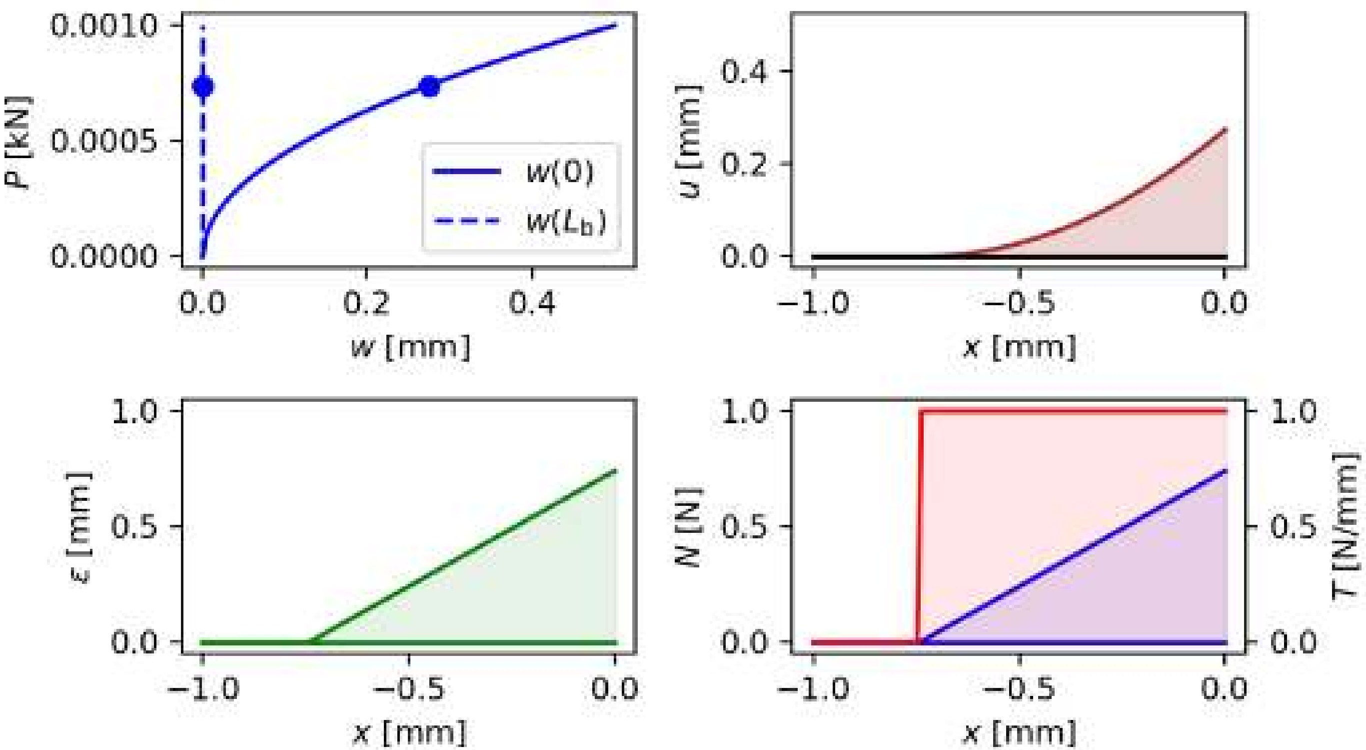
$$p = \bar{\tau} = E_\mathrm{f} = A_\mathrm{f} = 1$$

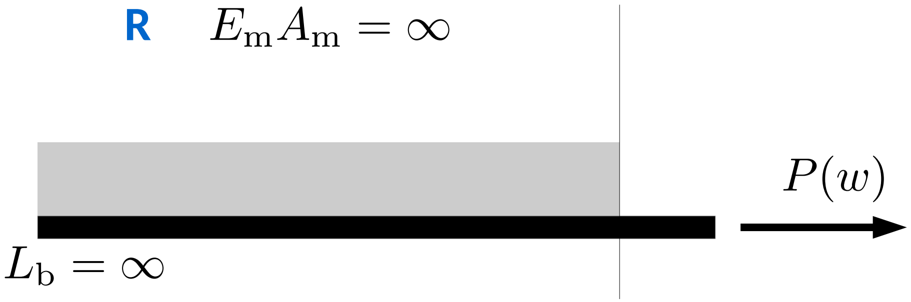


$$P(w) = \sqrt{2 p \bar{\tau} E_\mathrm{f} A_\mathrm{f} w}$$



$$w = \int_a^0 \varepsilon_\mathrm{f}(x)\,\mathrm{d}x$$

$$w = -\frac{1}{2}\varepsilon_\mathrm{f}(0)\,a$$



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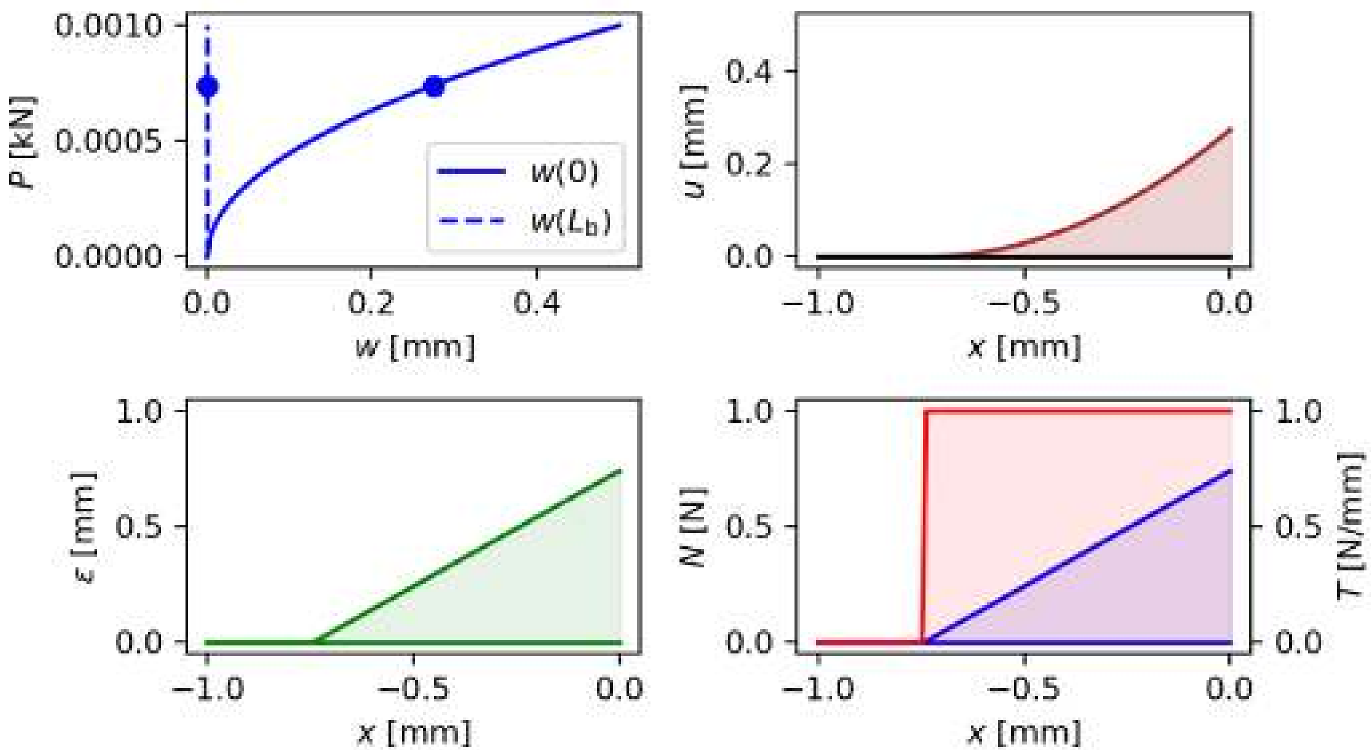
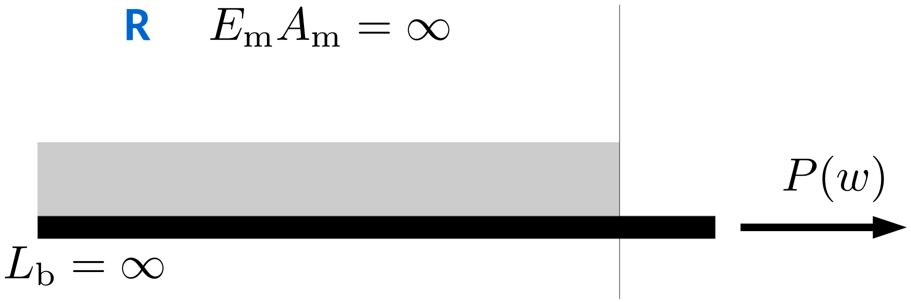


$$w = \int_a^0 \varepsilon_\mathrm{f}(x)\,\mathrm{d}x$$

$$\varepsilon_\mathrm{f}(0) = \frac{1}{E_\mathrm{f}} \sigma_\mathrm{f}(0) = \frac{P}{E_\mathrm{f} A_\mathrm{f}}$$

$$a = -\frac{P}{p\tau}$$

$$w = -\frac{1}{2} \varepsilon_\mathrm{f}(0)\, a$$



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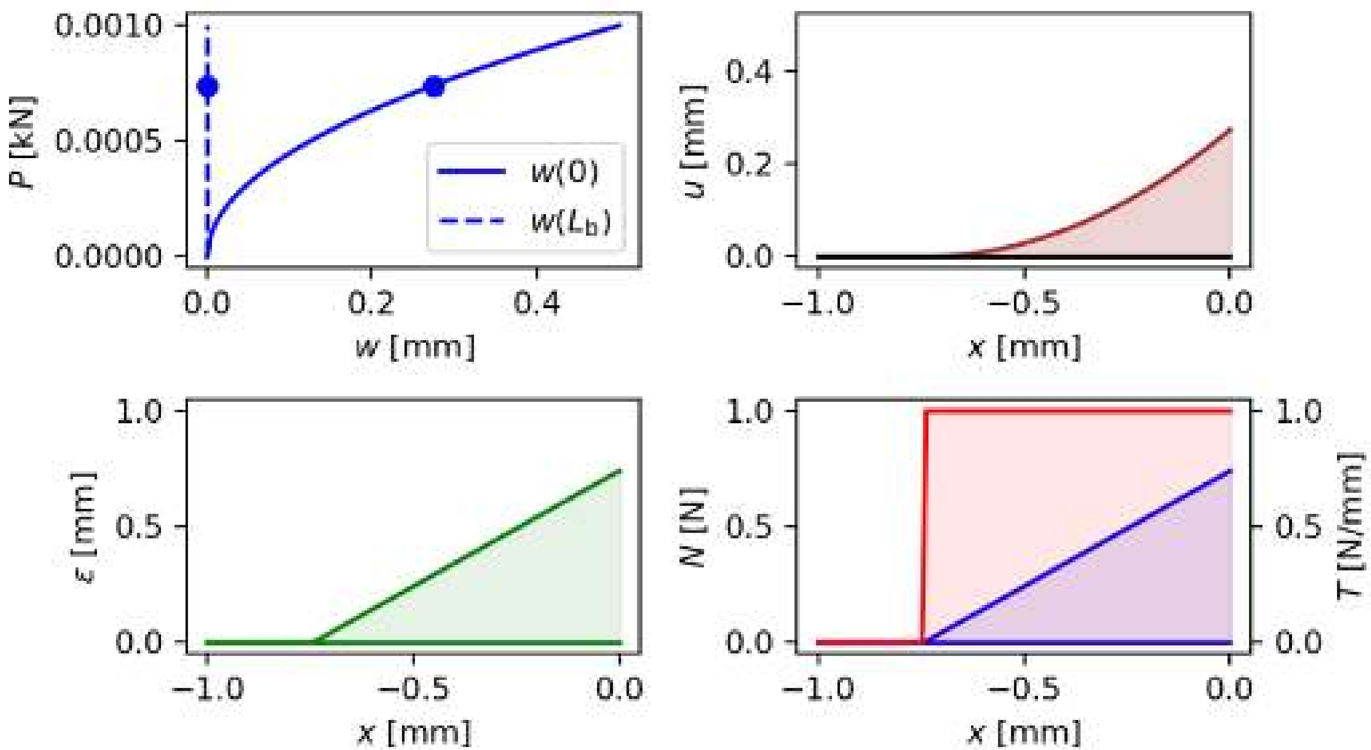

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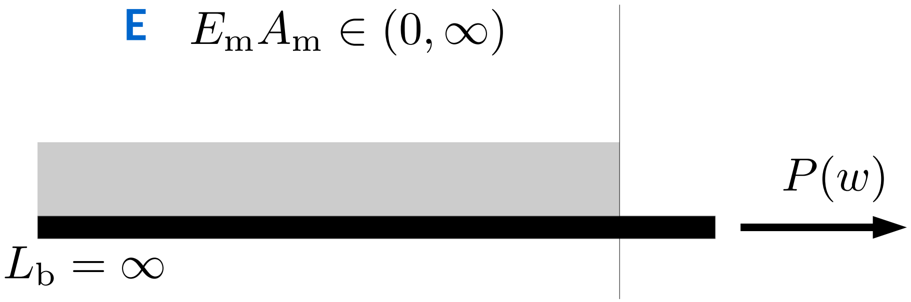
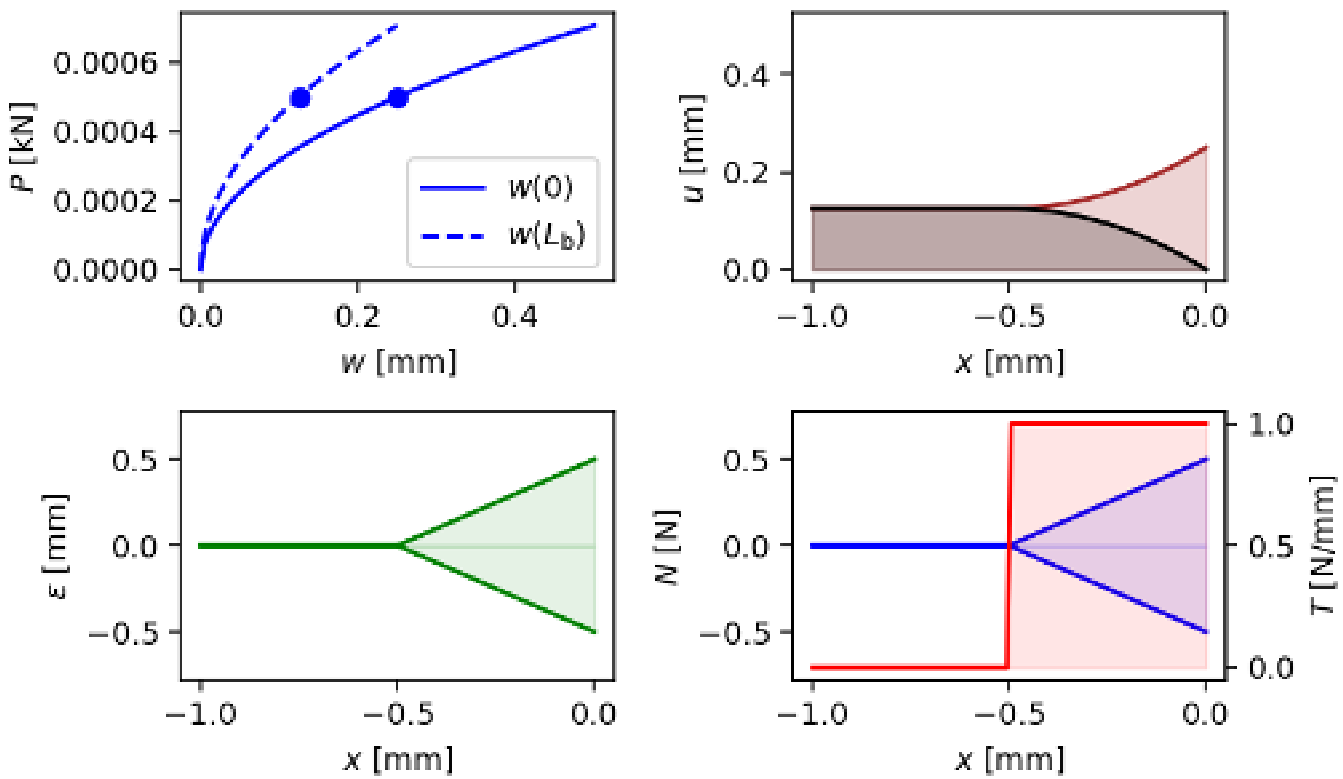
$$w = \frac{1}{2} \frac{P^2}{p\tau} \left[\frac{1}{E_\mathrm{f} A_\mathrm{f}} \right]$$

2.2 PO-ELF-ELM

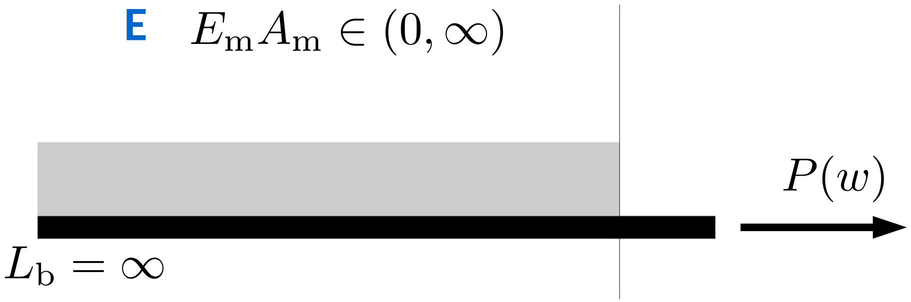


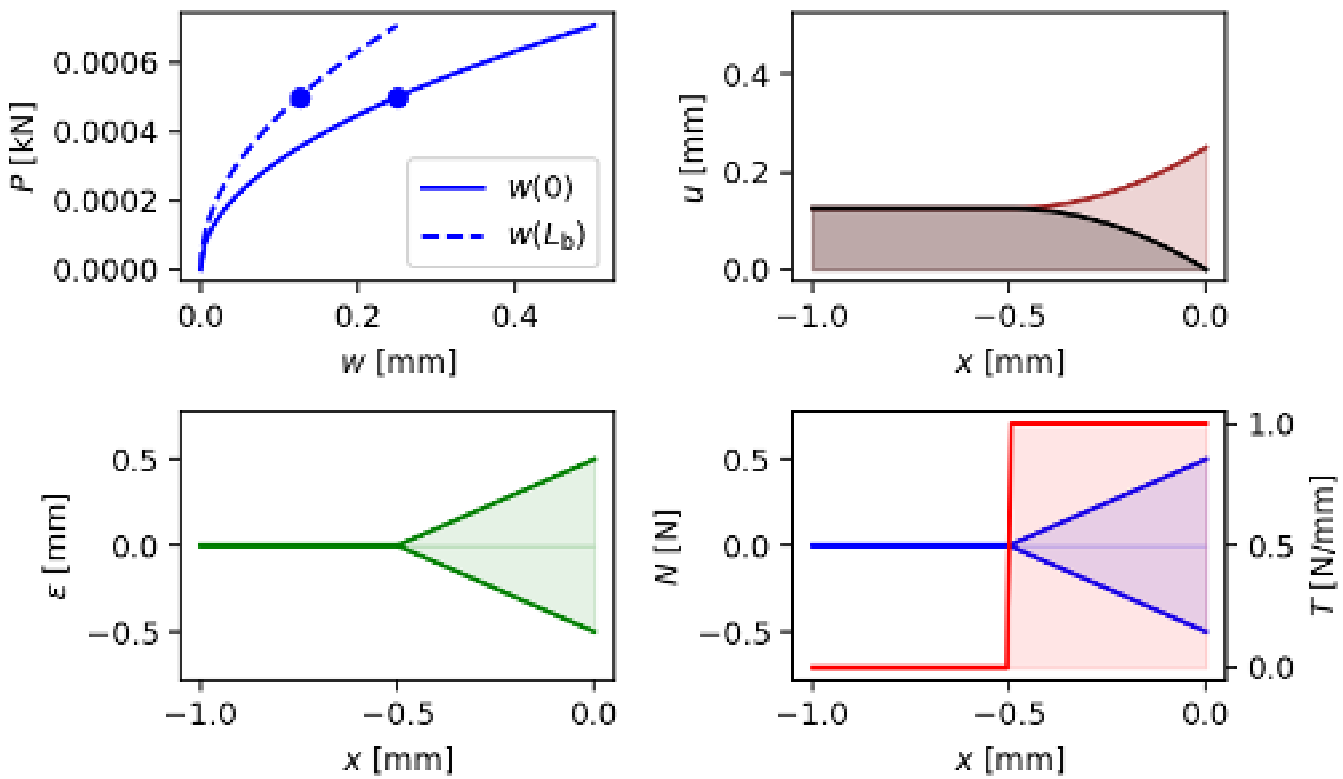


$$w = \int_a^0 \varepsilon_\mathrm{f} - \varepsilon_\mathrm{m} \, \mathrm{d}x$$

$$w = -\frac{1}{2}(\varepsilon_\mathrm{f}(0) - \varepsilon_\mathrm{m}(0))a$$

E $E_\mathrm{m} A_\mathrm{m} \in (0, \infty)$





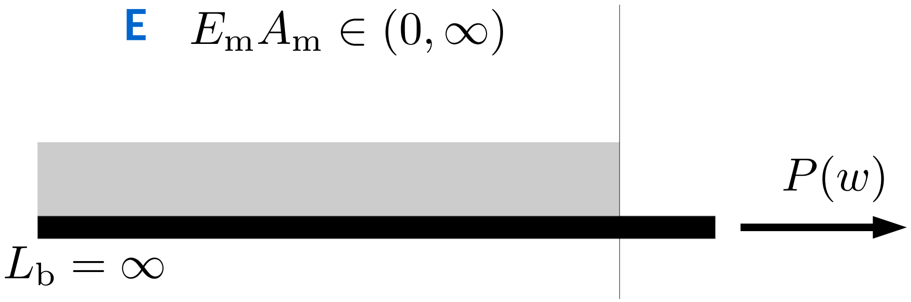
$$w = \int_a^0 \varepsilon_\mathrm{f} - \varepsilon_\mathrm{m} \, \mathrm{d}x$$

$$\varepsilon_\mathrm{f}(0) = \frac{P}{A_\mathrm{f} E_\mathrm{f}} \quad \varepsilon_\mathrm{m}(0) = -\frac{P}{A_\mathrm{m} E_\mathrm{m}}$$

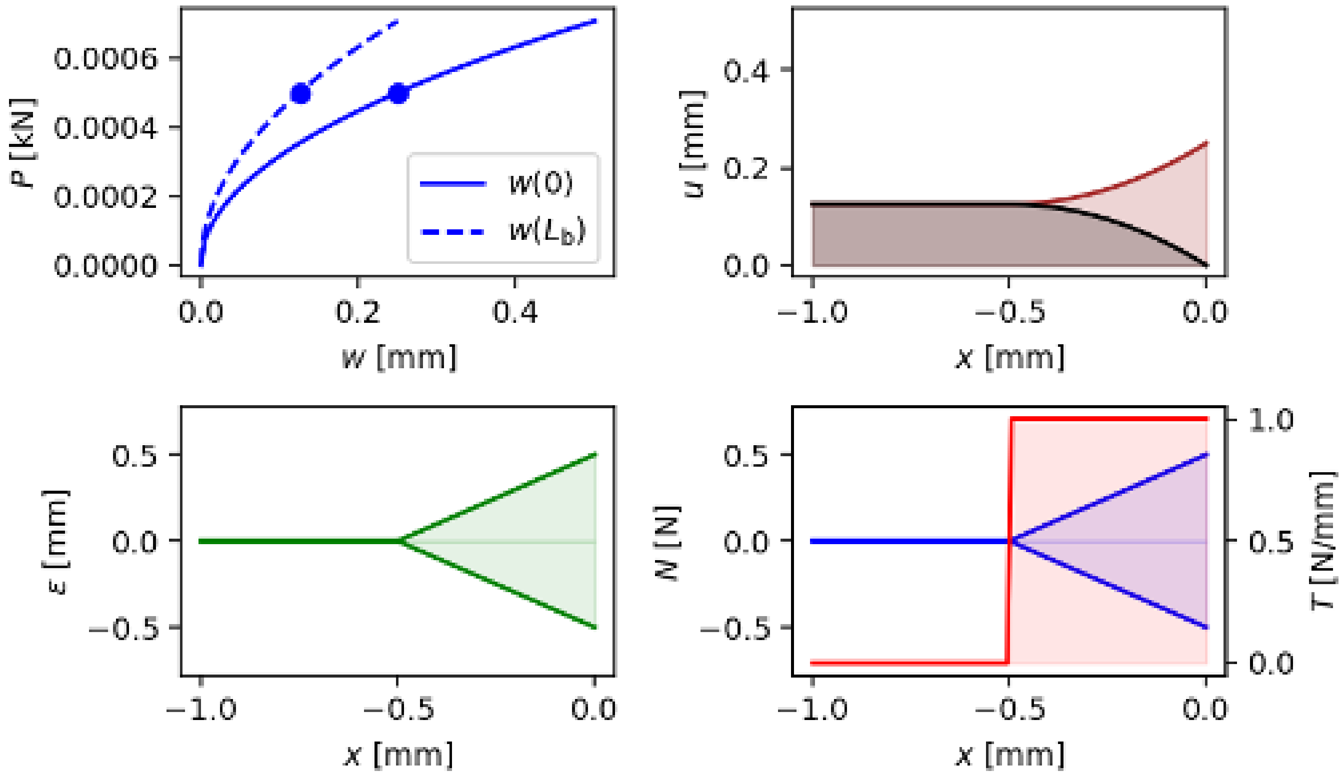
$$a = -\frac{P}{p\tau}$$

$$w = -\frac{1}{2}(\varepsilon_\mathrm{f}(0) - \varepsilon_\mathrm{m}(0))a$$

2.2 PO-ELF-ELM



$$P(w) = \sqrt{2wp\tau \frac{E_\mathrm{f} A_\mathrm{f} E_\mathrm{m} A_\mathrm{m}}{E_\mathrm{f} A_\mathrm{f} + E_\mathrm{m} A_\mathrm{m}}}$$



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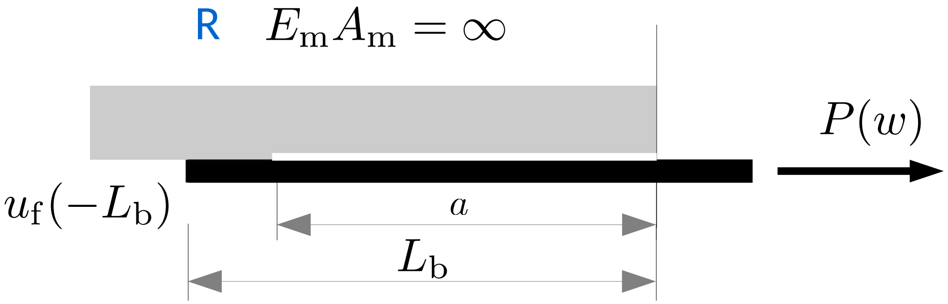
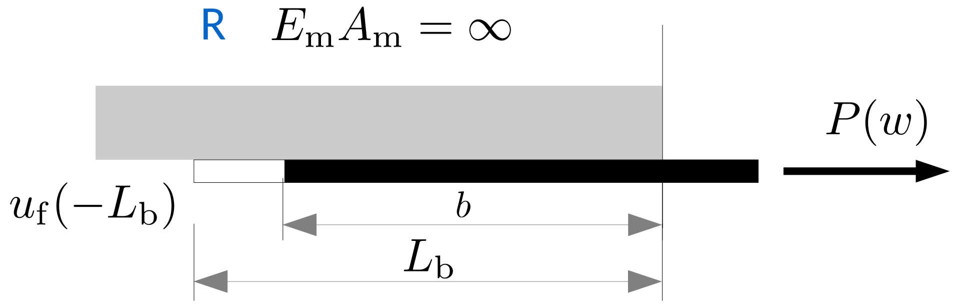
$$a = -\frac{P}{p\tau}$$

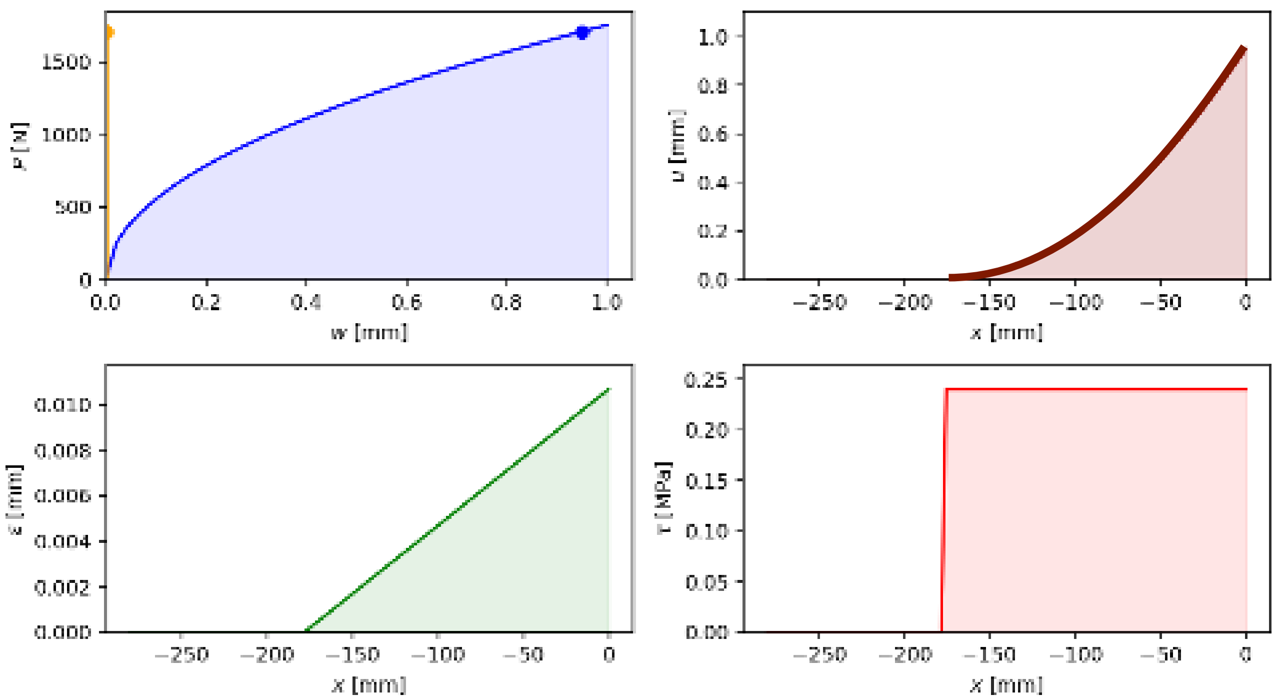
$$w = -\frac{1}{2}(\varepsilon_\mathrm{f}(0) - \varepsilon_\mathrm{m}(0))a$$

$$= \frac{1}{2} \frac{P^2}{p\tau} \left[\frac{1}{E_\mathrm{f} A_\mathrm{f}} + \frac{1}{E_\mathrm{m} A_\mathrm{m}} \right]$$

2.3 PO-ESF-RLM







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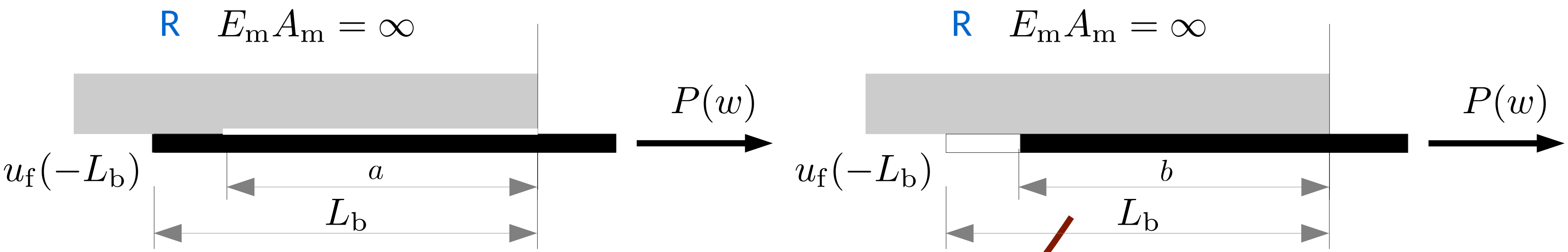

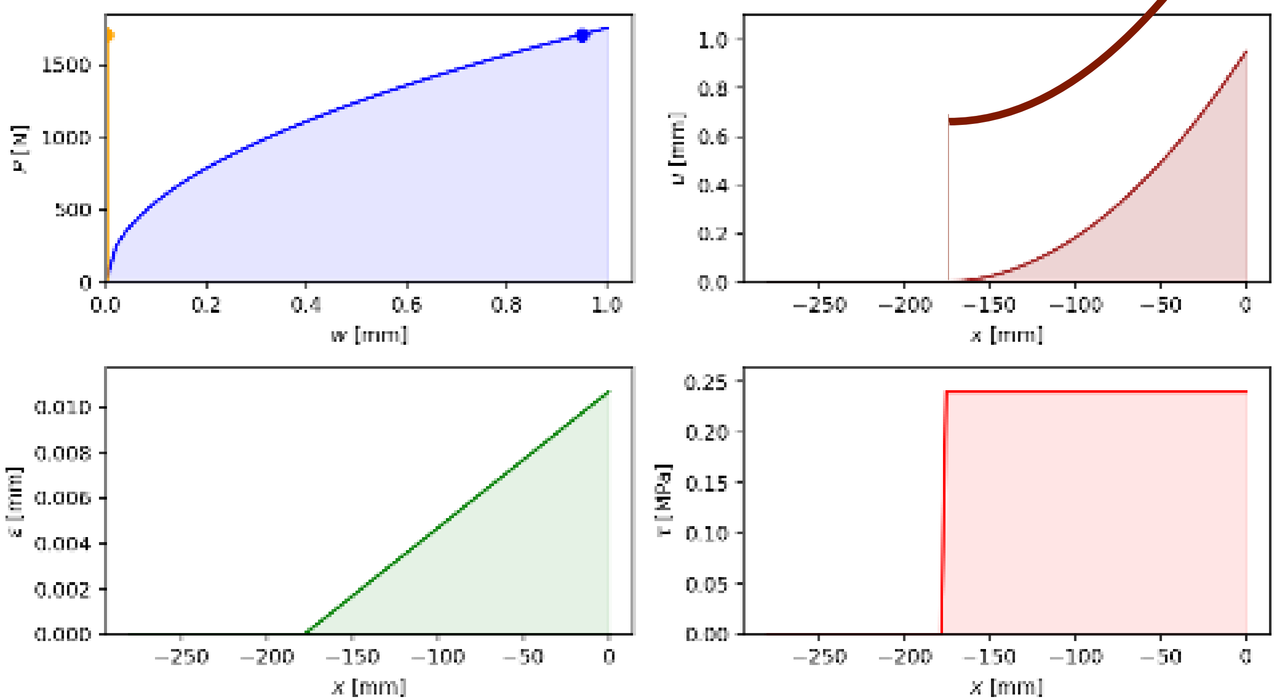

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Pullout with constant bond slip law – Takeaways

Wide range of pullout behavior can be achieved even for constant bond-slip law.
The response is piece-wise quadratic

Effect of modified boundary conditions?

Effect of model parameters?

Validity of the model – what kind of pull-out test can be predicted and what can not?

Pullout with constant bond slip law – Takeaways

Analytically derived models are focused or limited to a narrow range of parameters
specialized idealizations, relatively complex derivation of equations, support by math tools,
quick response – not only a simulator but also an optimization tool possible

See the jupyter notebooks and the corresponding videos
to see the interactively study the mentioned versions of the model
and to learn the mechanisms governing the pullout behavior

Guided tour 2 constant bond-slip behavior