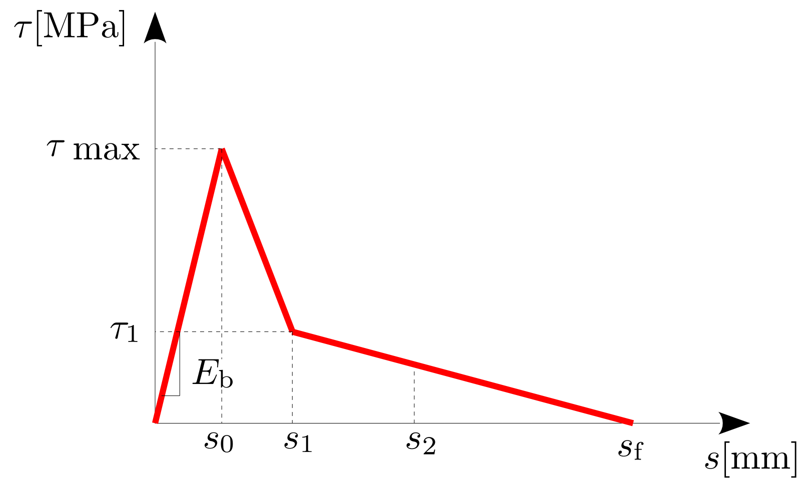
Bond-slip law expressed as damage function

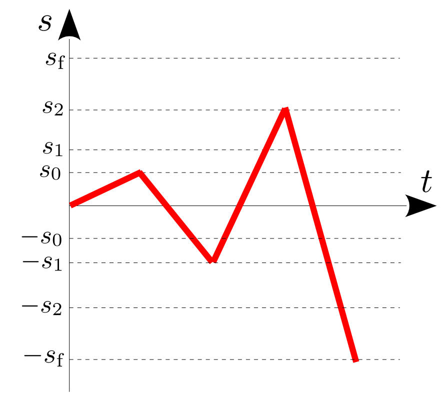
A tri-linear bond-slip law shown in the figure is given as follows

$$\tau(s) = \begin{cases} E_\mathrm{b}\, s & s \le s_0 \\ \tau_\mathrm{max} + (\tau_1 - \tau_\mathrm{max})\left(\dfrac{s - s_0}{s_1 - s_0}\right) & s_0 < s \le s_1 \\ \tau_1 - \tau_1\left(\dfrac{s - s_1}{s_\mathrm{f} - s_1}\right) & s_1 < s \le s_\mathrm{f} \end{cases}$$



Assuming that the bond behavior is governed purely by damage and provided the characteristic points of the bond-slip law are given as

τ_1 = 5.0 MPa, s_0 = 0.1 mm, s_1 = 0.2 mm, s_f = 1.0 mm, Eb = 100 MPa/mm



a) Derive a damage function which would reproduce the given bond-slip law
b) Calculate the damage and stiffness values at the slip s_1.
c) Sketch graphically the derived damage function in a damage-slip diagram.
d) Plot the response of a material point exposed to the shown loading history

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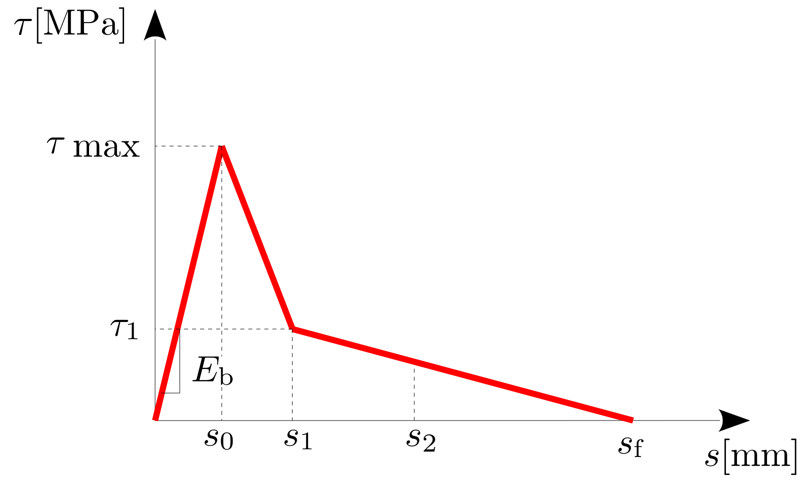
$\tau_1 = 5.0$ MPa, $s_0 = 0.1$ mm, $s_1 = 0.2$ mm,
$s_f = 1.0$ mm, Eb $= 100$ MPa/mm



a) Derive a damage function which would reproduce the given bond-slip law.

Solution:

$$\tau(s) = [1 - \omega(s)]\ E_\mathrm{b}\ s \quad \Rightarrow \quad \omega(s) = 1 - \frac{\tau(s)}{E_\mathrm{b}\ s}$$

e.g. range 1 ($s \leq s_0$) : $$\omega(s) = 1 - \frac{E_\mathrm{b}\ s}{E_\mathrm{b}\ s} \quad \Rightarrow \quad \omega(s) = 0$$

$$\omega(s) = \begin{cases} 0 & s \leq s_0 \\ 1 - \frac{1}{E_\mathrm{b}\ s}\left[\tau_\mathrm{max} + (\tau_1 - \tau_\mathrm{max})(\frac{s - s_0}{s_1 - s_0})\right] & s_0 < s \leq s_1 \\ 1 - \frac{1}{E_\mathrm{b}\ s}\left[\tau_1 - \tau_1(\frac{s - s_1}{s_\mathrm{f} - s_1})\right] & s_1 < s \leq s_\mathrm{f} \end{cases}$$

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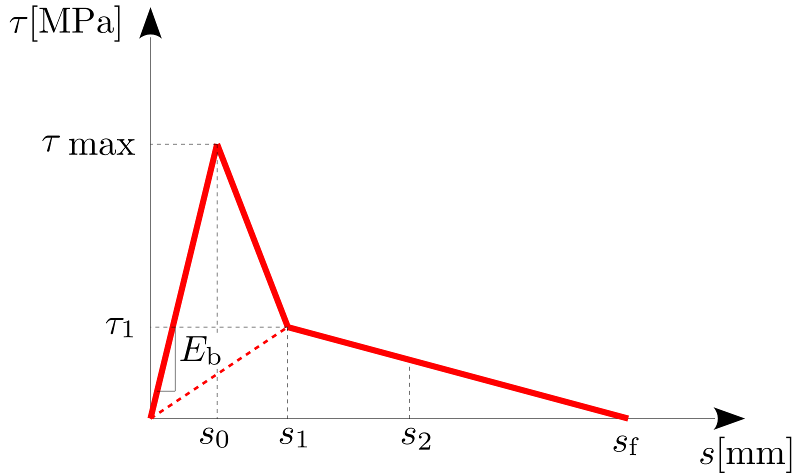

b) Calculate the damage and stiffness values at the slip s_1.

Solution:

Damage:

$$\omega(s) = 1 - \frac{1}{E_\mathrm{b}\ s}\left[\tau_\mathrm{max} + (\tau_1 - \tau_\mathrm{max})(\frac{s - s_0}{s_1 - s_0})\right] \quad s_0 < s \leq s_1$$

$$\Rightarrow \omega(s = 0.2) = 1 - \frac{1}{100 \times 0.2}\left[10 + (5 - 10)(\frac{0.2 - 0.1}{0.2 - 0.1})\right] = 0.75$$

Stiffness:

$$E(\omega) = (1 - \omega)E_\mathrm{b}$$

$$\Rightarrow E(\omega = 0.75) = (1 - 0.75)100 = 25\ [\mathrm{MPa/mm}]$$

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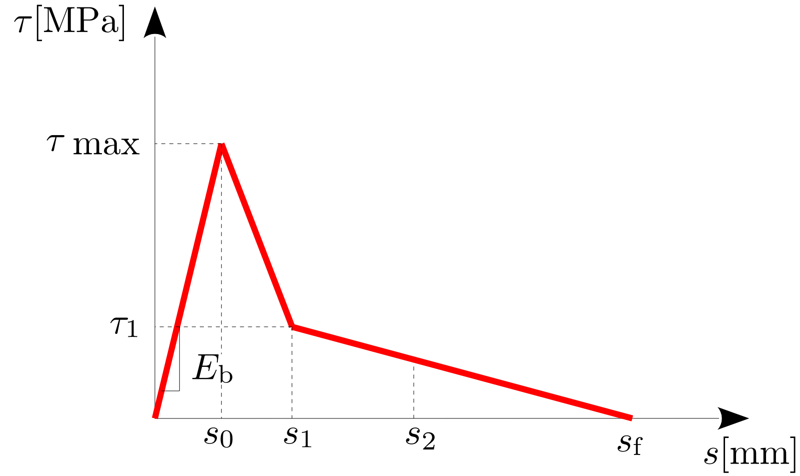


c) Sketch graphically the derived damage function in a damage-slip diagram.

Solution:

at s_0 $\quad \omega(s = s_0 = 0.1) = 0 \qquad$ (elastic range)

at s_1

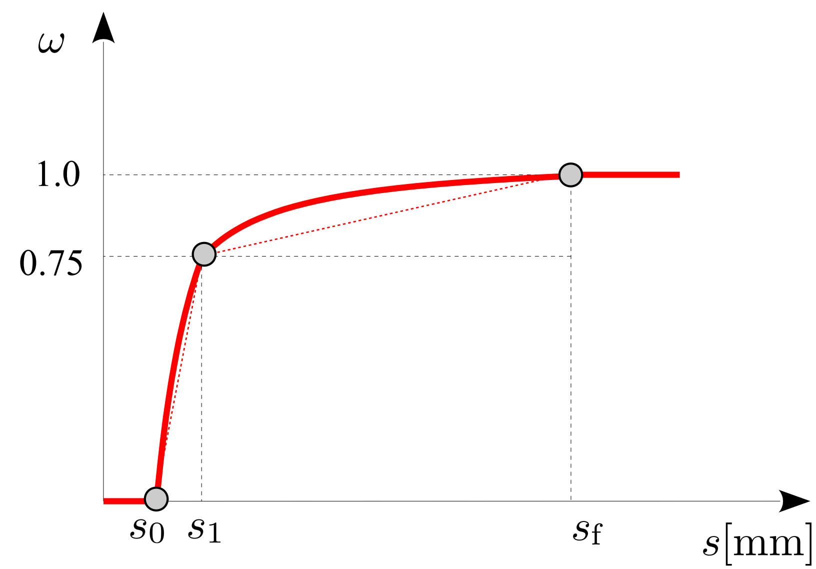
$$\omega(s) = 1 - \frac{1}{E_\mathrm{b}\, s}\left[\tau_\mathrm{max} + (\tau_1 - \tau_\mathrm{max})(\frac{s - s_0}{s_1 - s_0})\right] \quad s_0 < s \leq s_1$$

$$\omega(s = 0.2) = 1 - \frac{1}{100 \times 0.2}\left[10 + (5 - 10)(\frac{0.2 - 0.1}{0.2 - 0.1})\right] = 0.75$$

at s_f

$$\omega(s) = 1 - \frac{1}{E_\mathrm{b}\, s}\left[\tau_1 - \tau_1(\frac{s - s_1}{s_\mathrm{f} - s_1})\right]$$

$$\omega(s = 1.0) = 1 - \frac{1}{100 \times 1.0}\left[5 - 5(\frac{1.0 - 0.2}{1.0 - 0.2})\right] = 1.0$$



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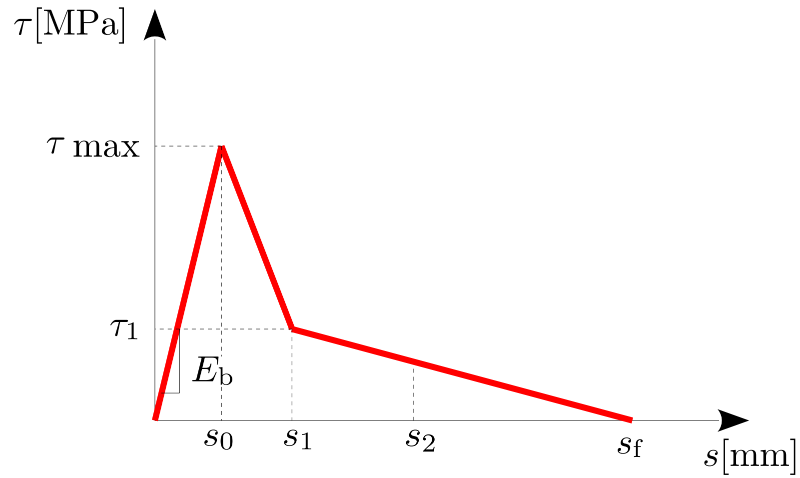

d) Plot the response of a material point exposed to the shown loading history.