bmcs course

Energy Games

work supply, stored energy, energy dissipation

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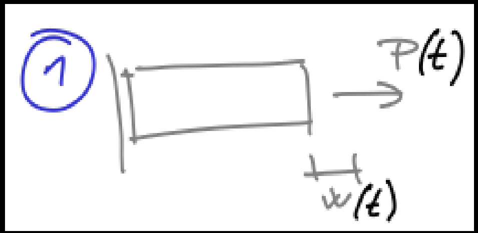
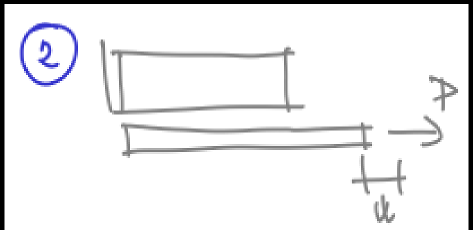
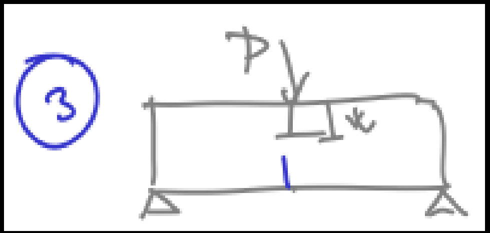
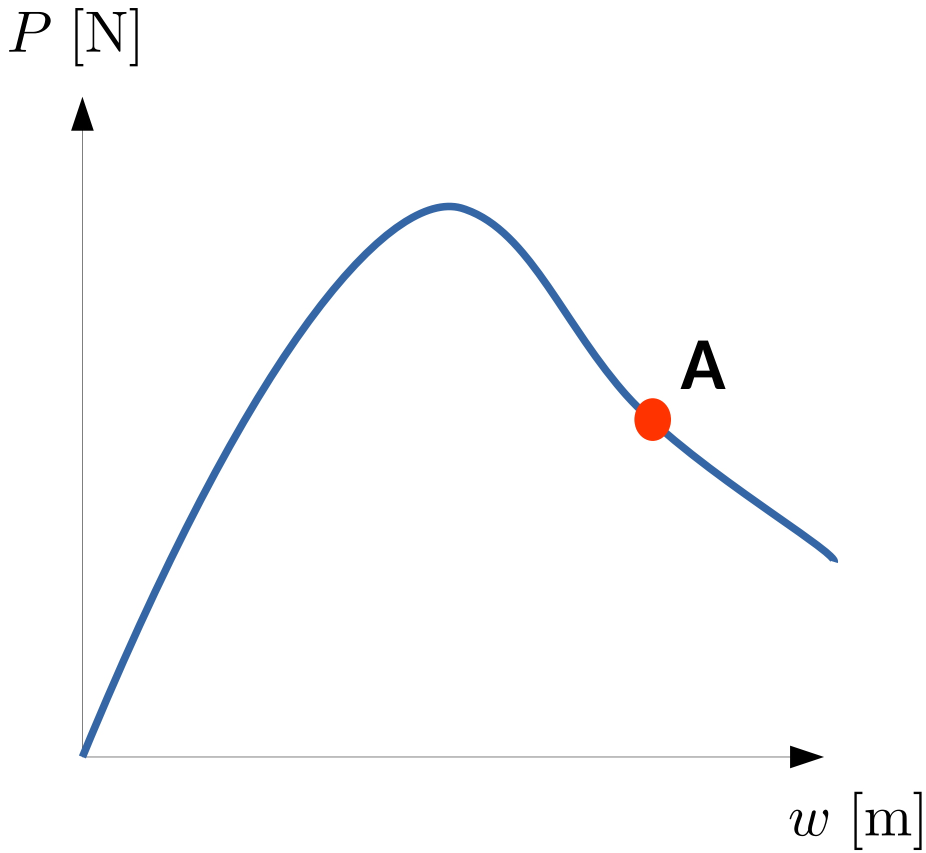
work supply







There is even more information hidden in the pull-out curve



How much energy do we have to supply to reach the point A?

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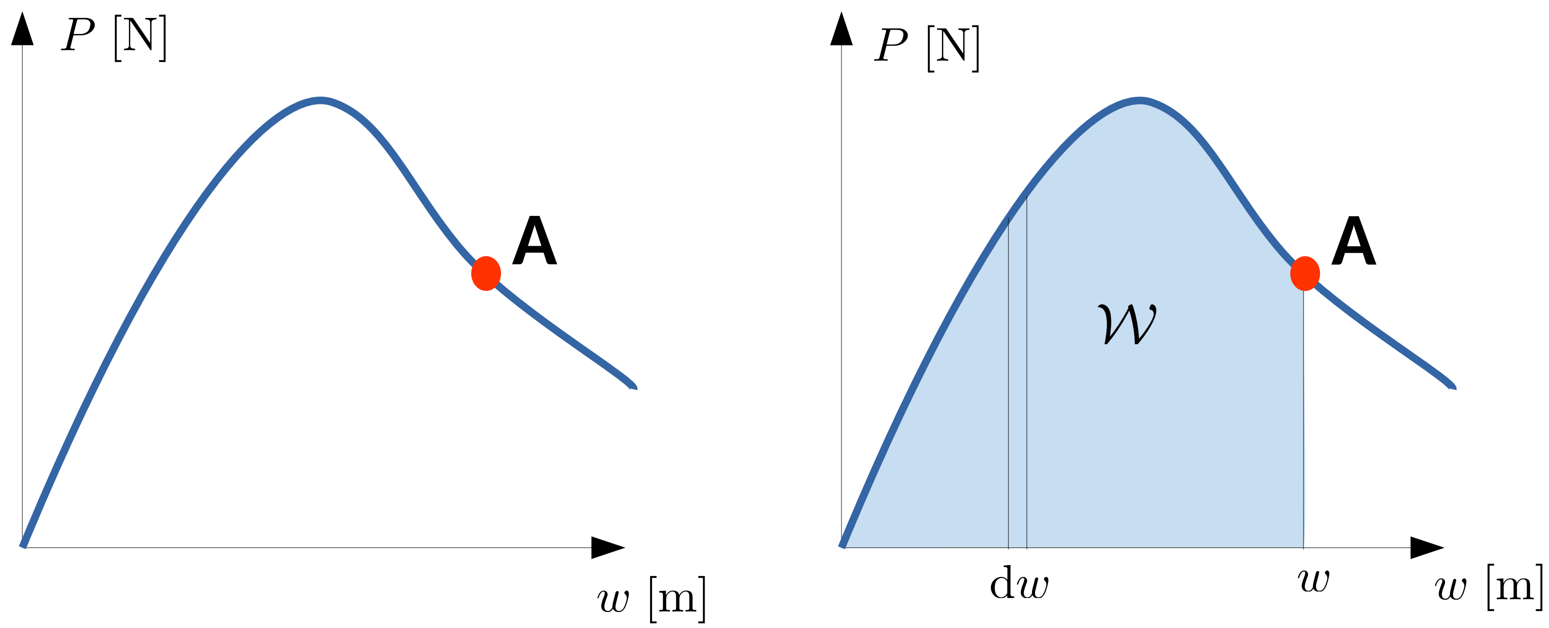

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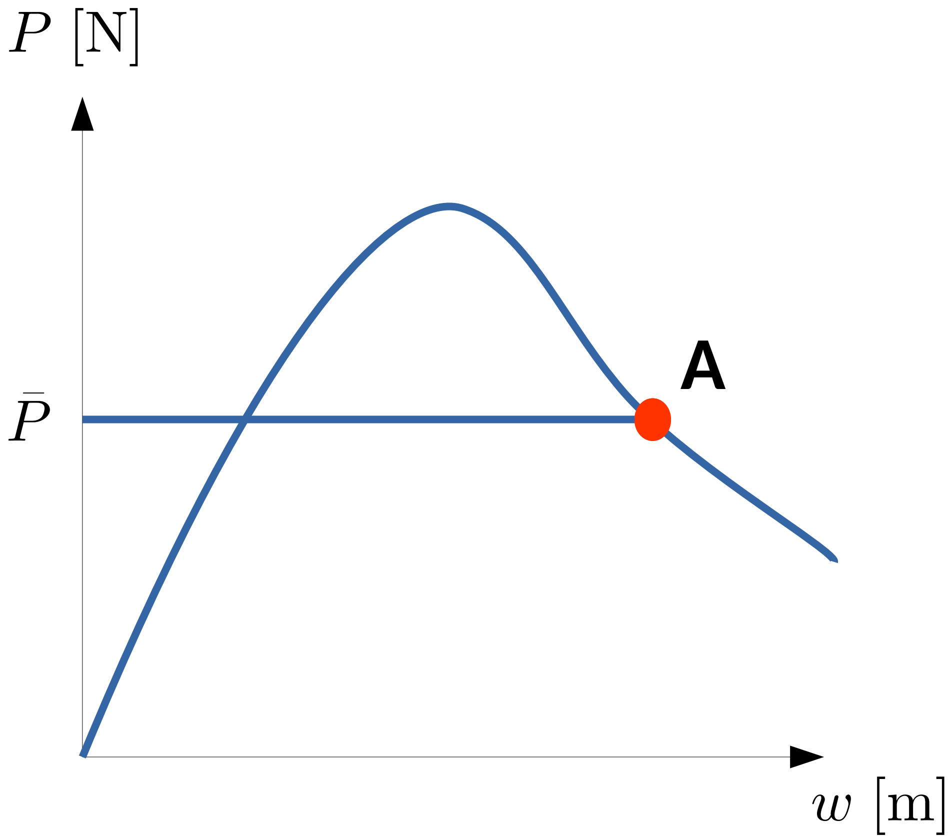

How much energy do we have to supply to reach the point A?

Recall the work definition: assuming constant force $\bar{P}$ [N] the work needed to displace an object to w [m] is

$$W = \bar{P} w \; [\mathrm{Nm}]$$

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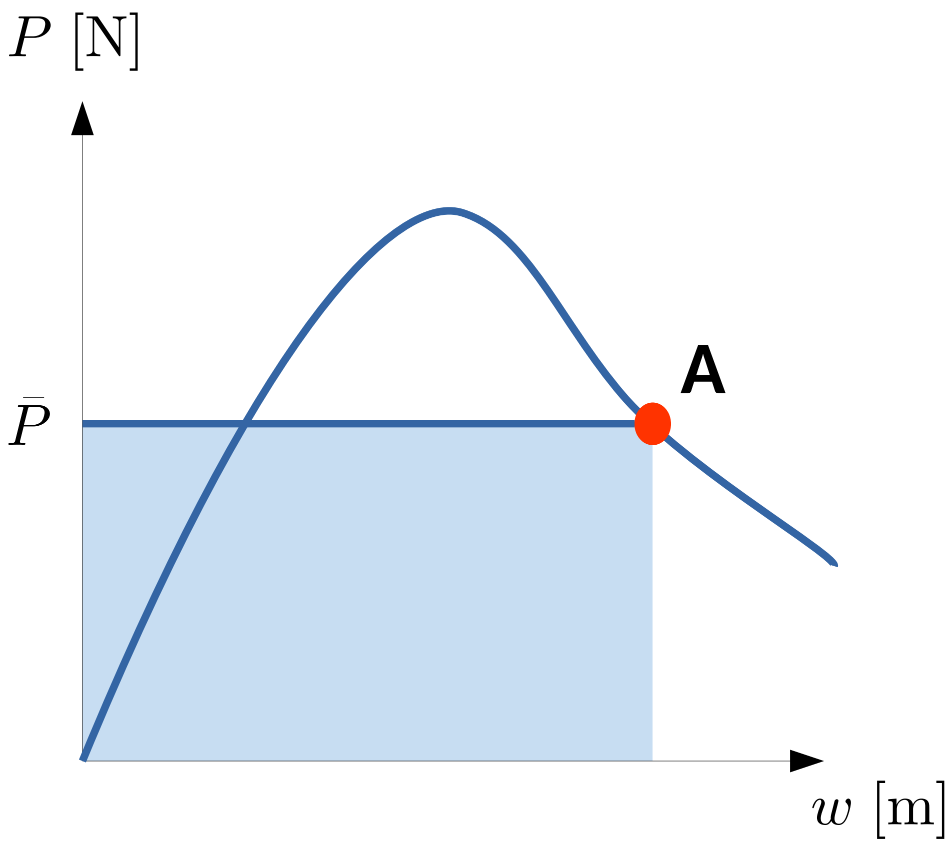

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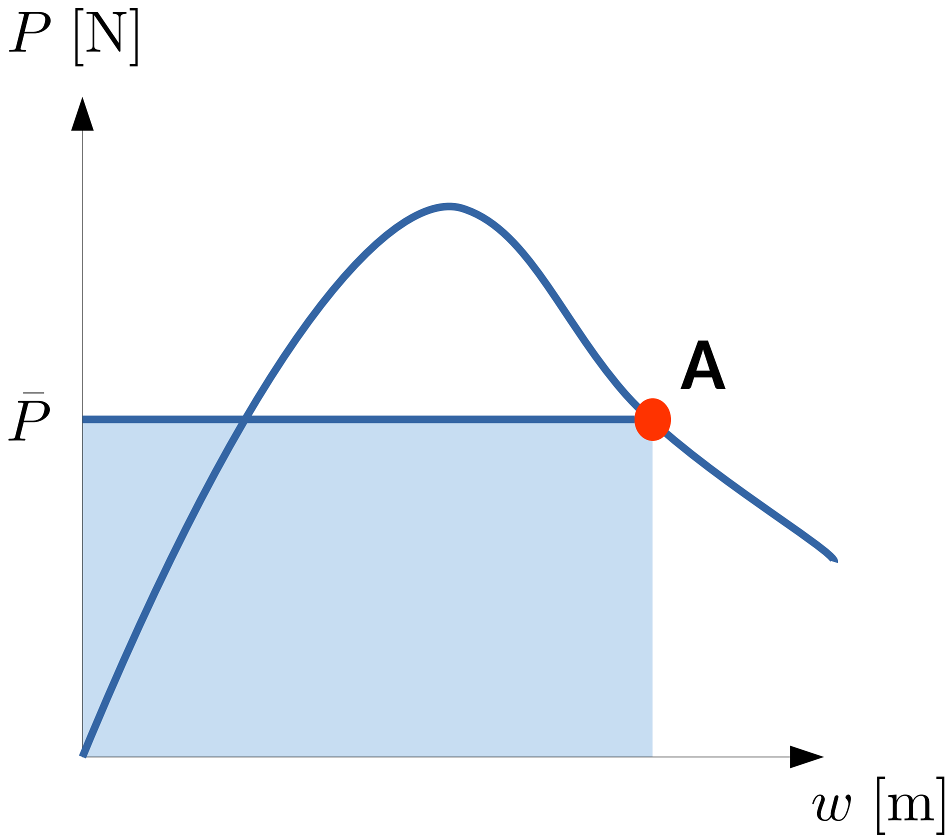
Work supply



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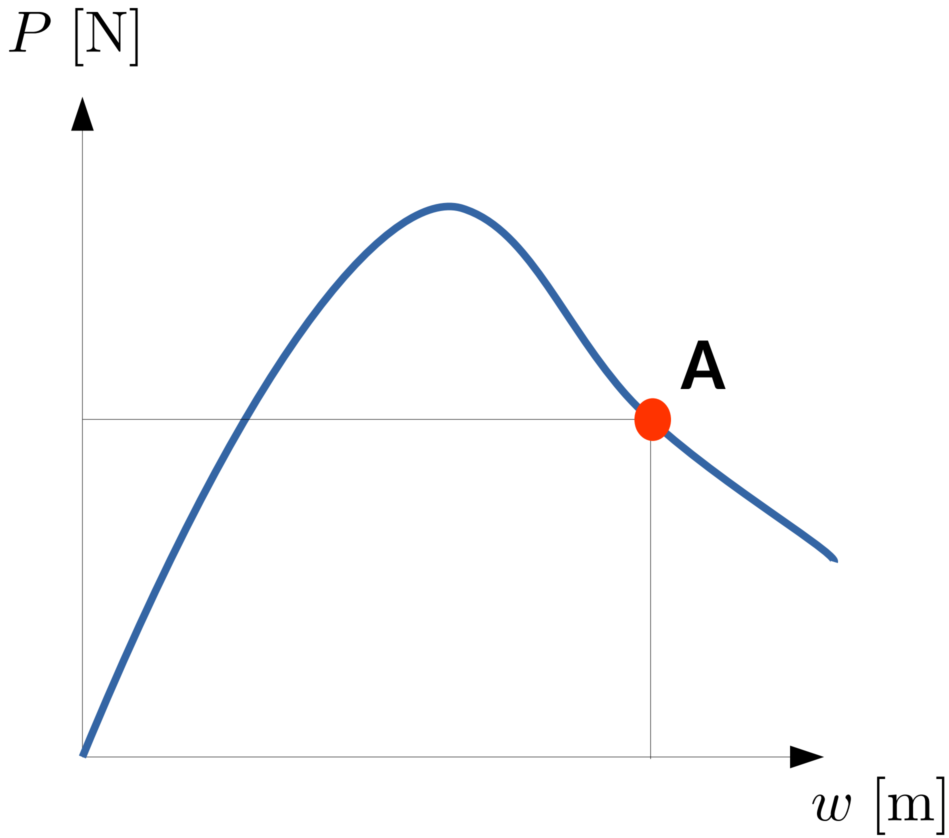
BUT what if $P(w) \neq \text{constant}$

Work supply



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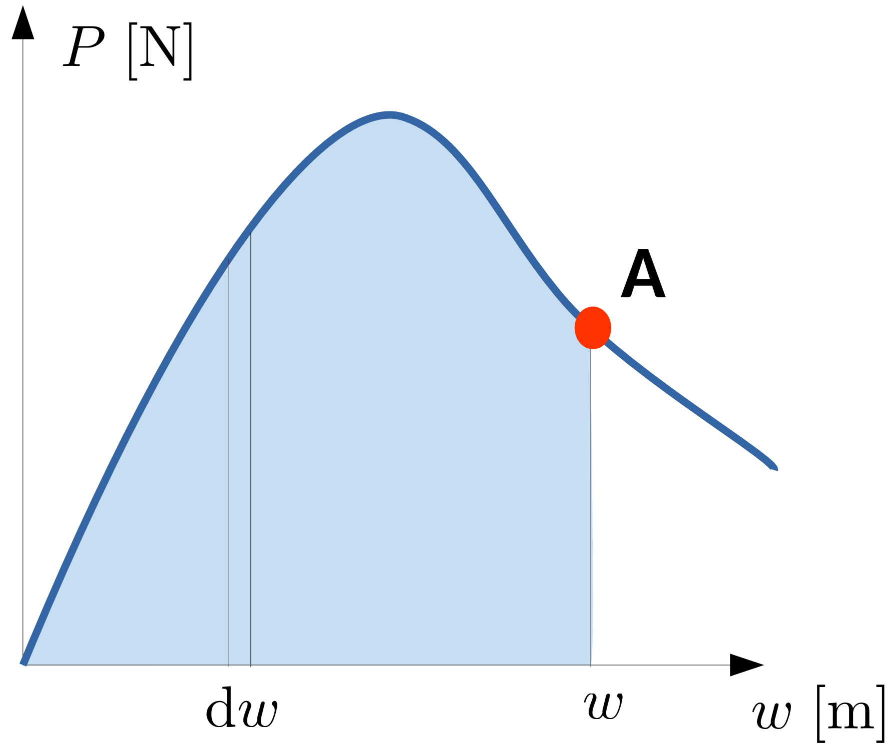
BUT what if $P(w) \neq \text{constant}$

THEN $\mathrm{d}W = P(w)\,\mathrm{d}w$

Work supply



How much energy do we have to supply to reach the point A?

$$\mathrm{d}W = P(w)\,\mathrm{d}w$$

$$W = \int_0^w P(w)\,\mathrm{d}w$$

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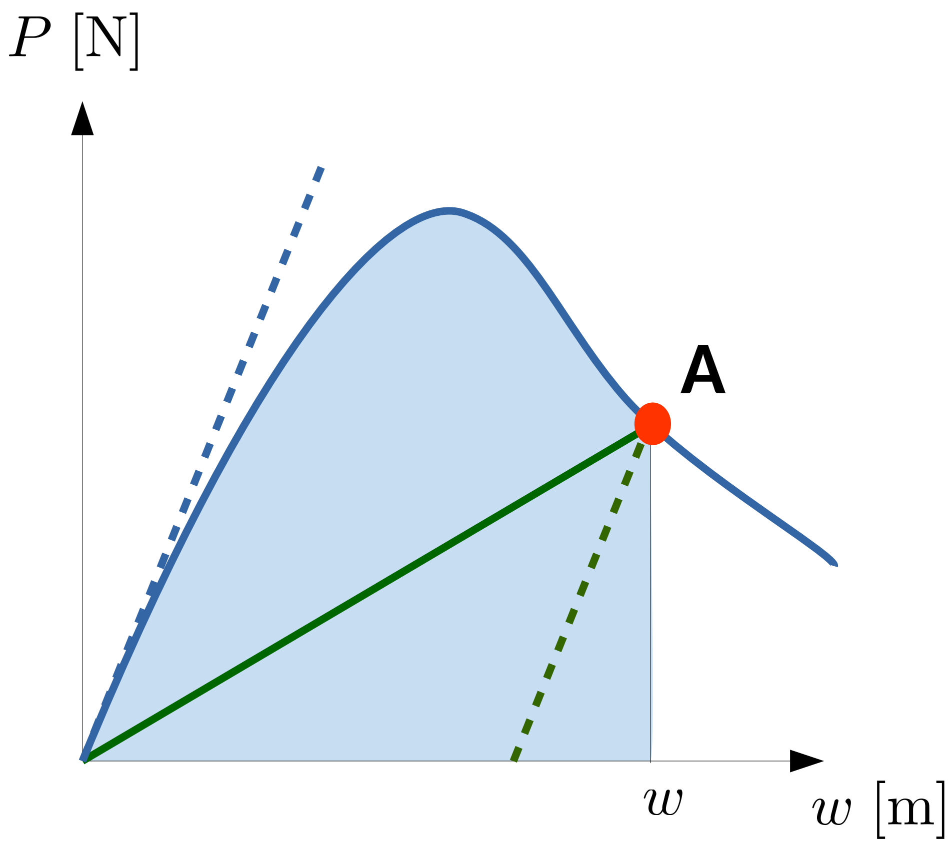
the battery – stored energy

Stored energy



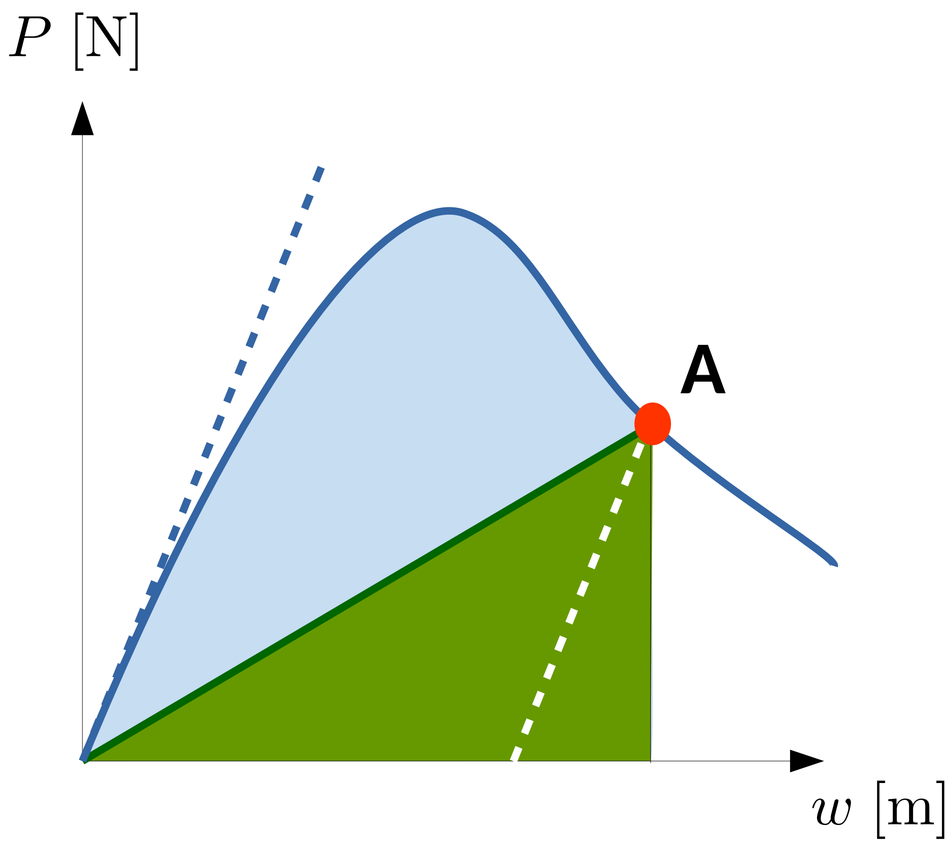
How much energy can be recovered by unloading?

It depends ...

Are you asking this for material behavior governed by damage or by plasticity?

$$W = \int_0^w P(w)\,\mathrm{d}w$$

Stored energy



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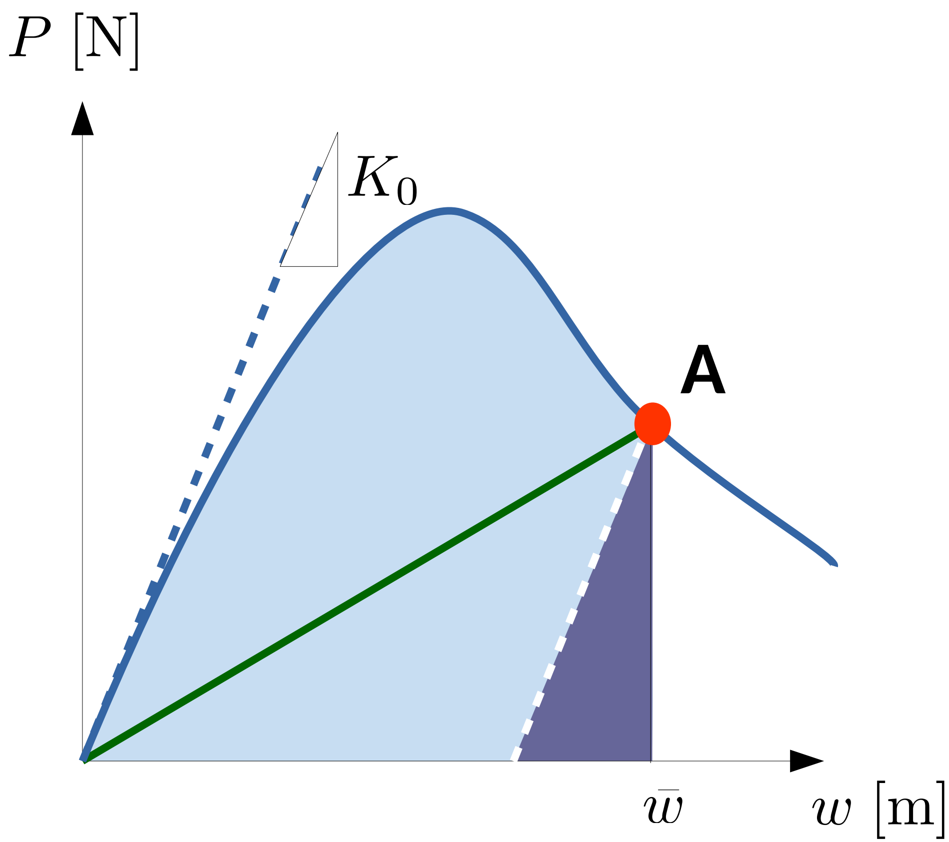
damage

$$U_\omega = \frac{1}{2} P w$$

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w$$

Stored energy



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damage

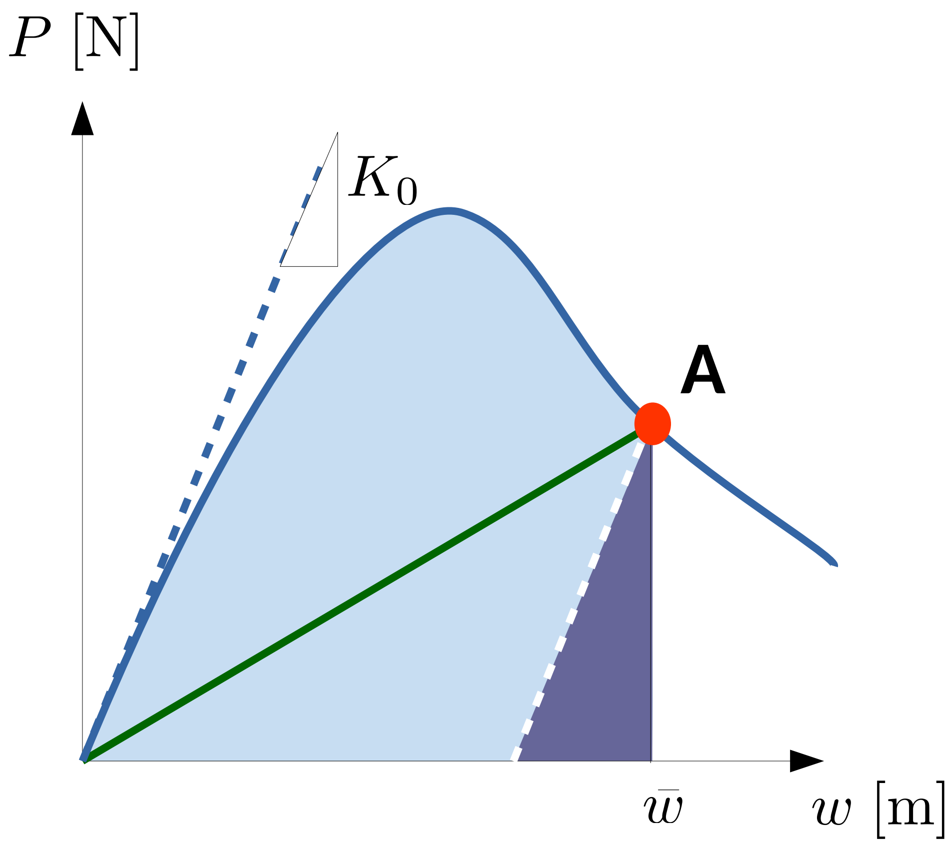
$$U_\omega = \frac{1}{2} P w$$

plasticity

$$U_\pi = \frac{1}{2} P \left(\frac{P}{K_0} \right)$$

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w$$

Stored energy



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damage

$$U_\omega = \frac{1}{2} P w$$

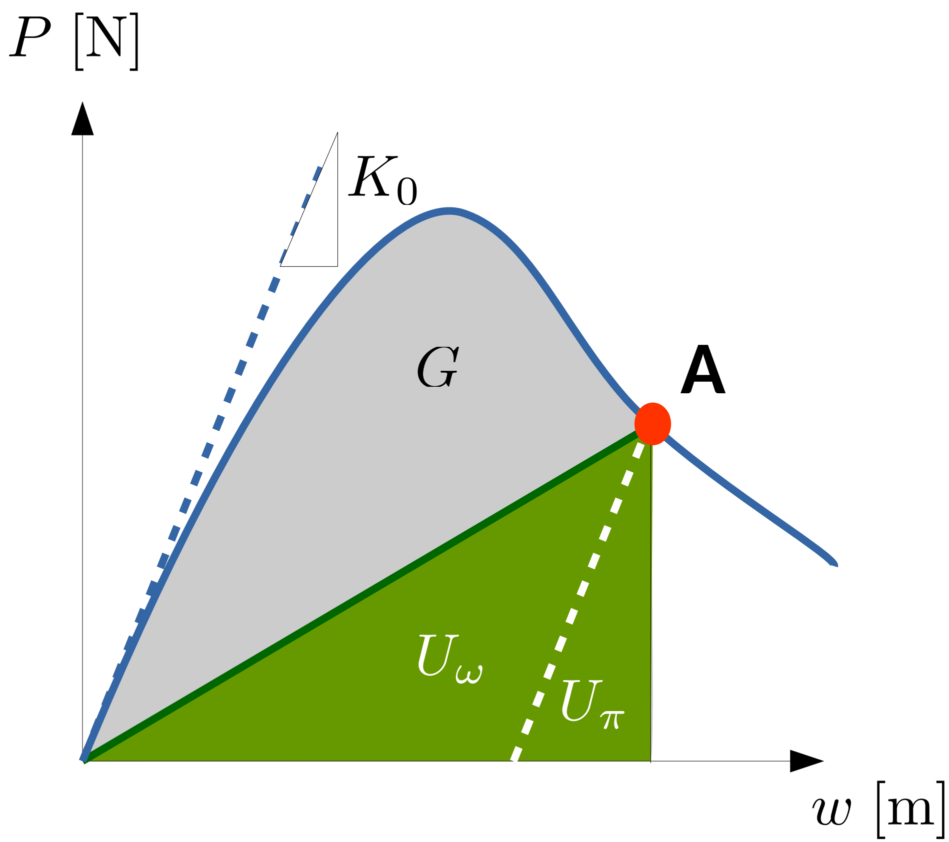
plasticity

$$U_\pi = \frac{P^2}{2K_0}$$

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0}$$

the energy release

Dissipated energy



How big was the energy leak?

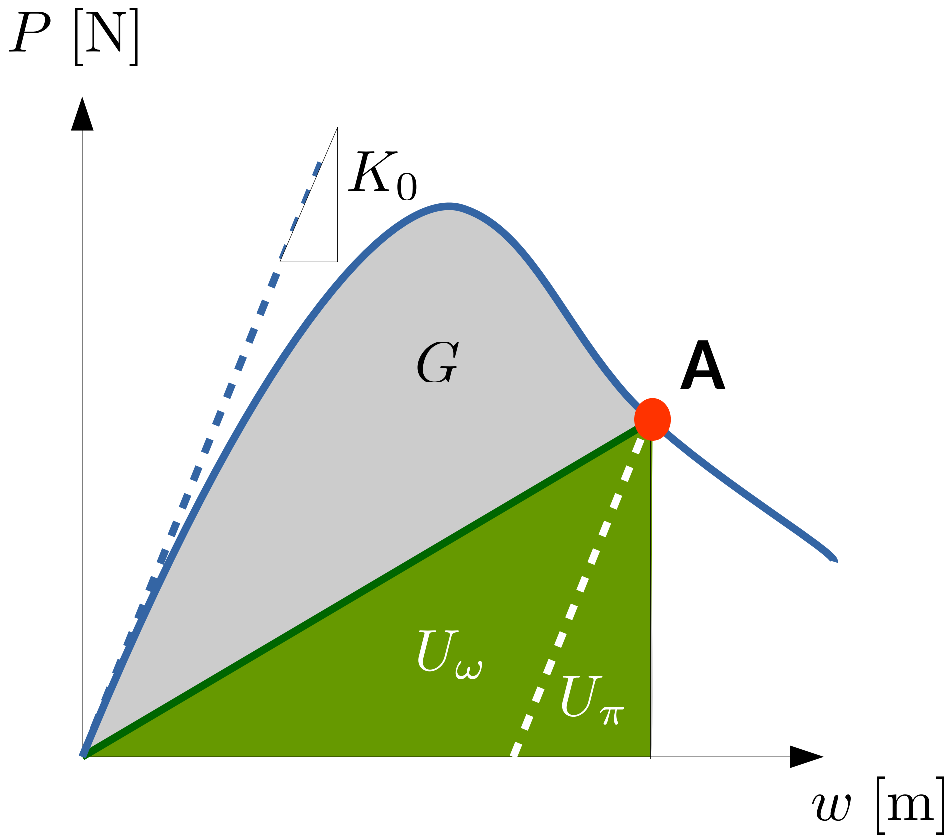
$$W(t) \longrightarrow \boxed{\Omega} \longrightarrow U(t)$$

$$G(t) = \underline{W}(t) - \underline{U}(t)$$

$$G = W - U = \int_0^w P(w)\,\mathrm{d}w - U(w)$$

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2}Pw \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U$$

Dissipated energy



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δ
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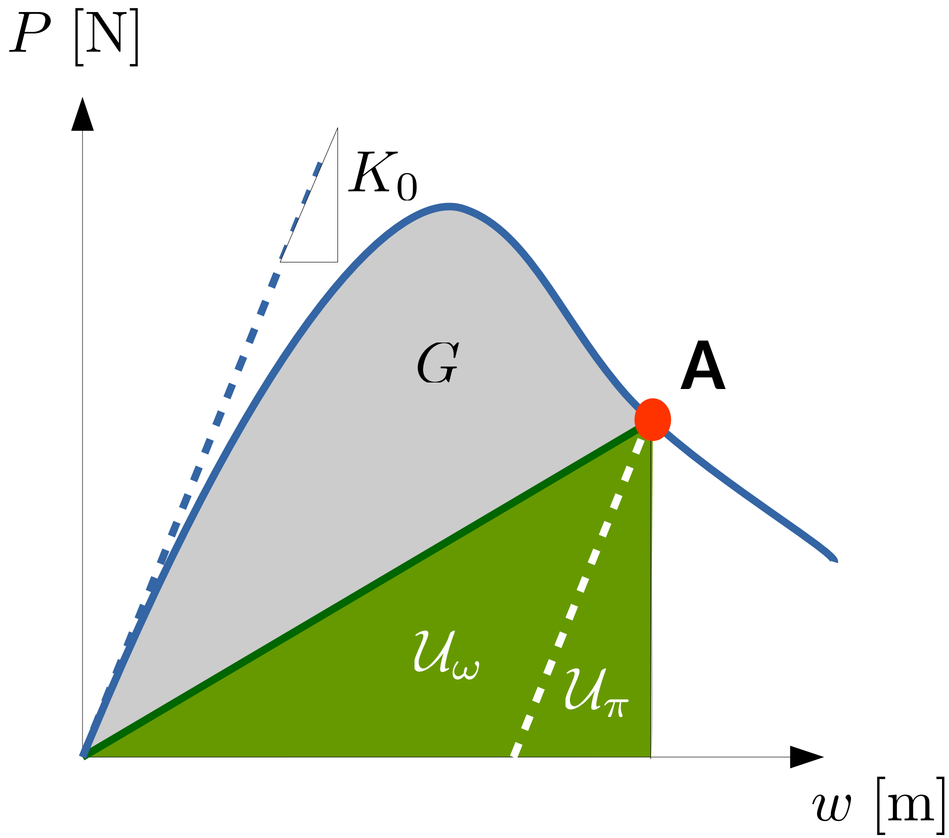
material behavior, boundary conditions, geometry

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2}Pw \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U$$

Dissipated energy



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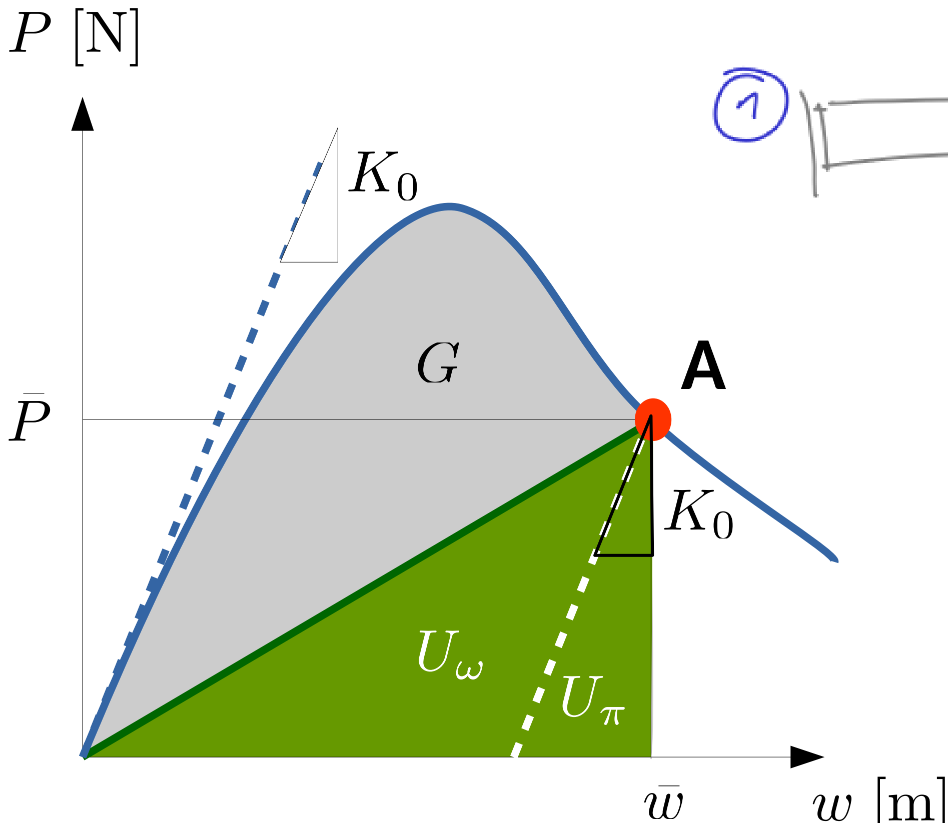
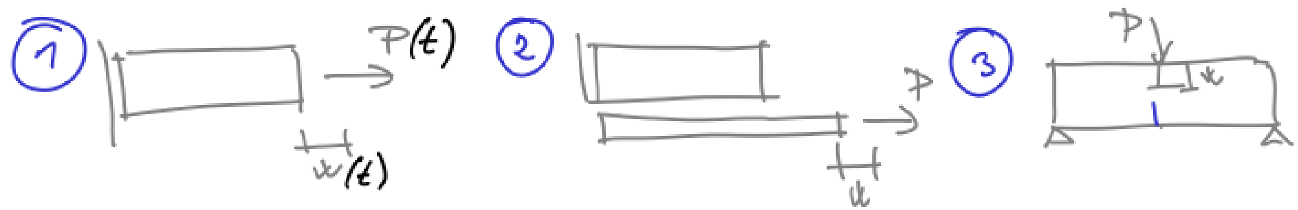
material behavior, boundary conditions, geometry

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2}Pw \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U$$

Dissipated energy



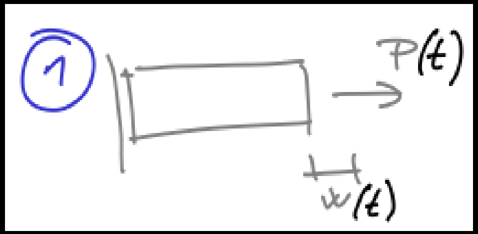


$$G = W - U = \int_0^w P(w)\,\mathrm{d}w - U(w)$$

material behavior, boundary conditions, geometry

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2}Pw \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U$$

elastic bar

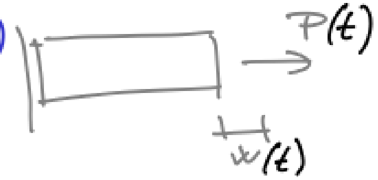


$$G = ?$$

Dissipated energy – elastic bar loaded in tension

$$G = W - U = \int_0^w P(w)\,\mathrm{d}w - U(w)$$

1



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Dissipated energy – elastic bar loaded in tension

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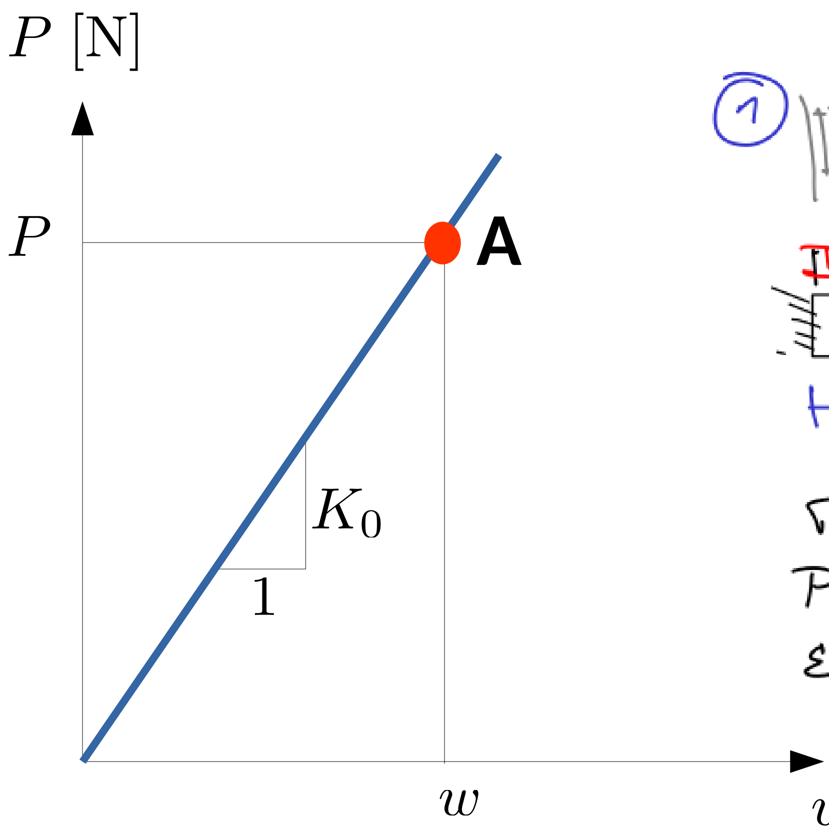

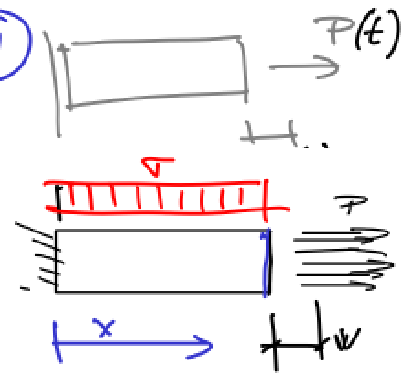

$\sigma = E\varepsilon$ – const. law

$P = A\sigma$ – equilibrium

$\varepsilon = \dfrac{w}{L}$ - kinematics

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Dissipated energy – elastic bar loaded in tension

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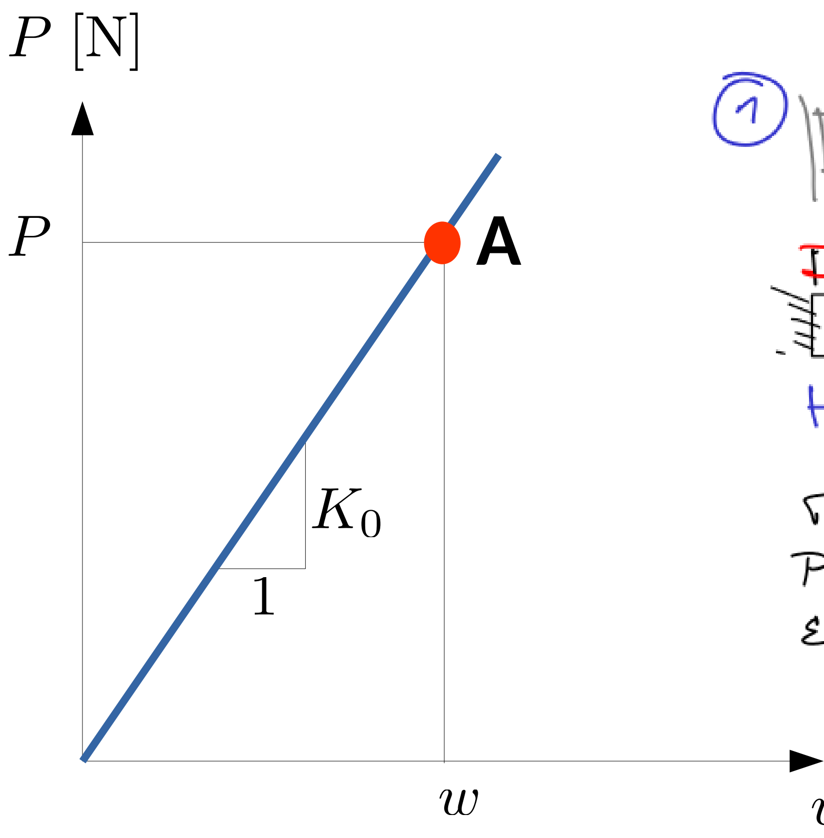

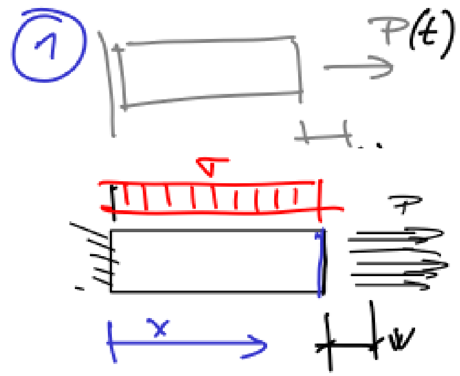

$$W = \int_0^w \frac{EA}{L} w\,\mathrm{d}w = \frac{EA}{2L} w^2$$

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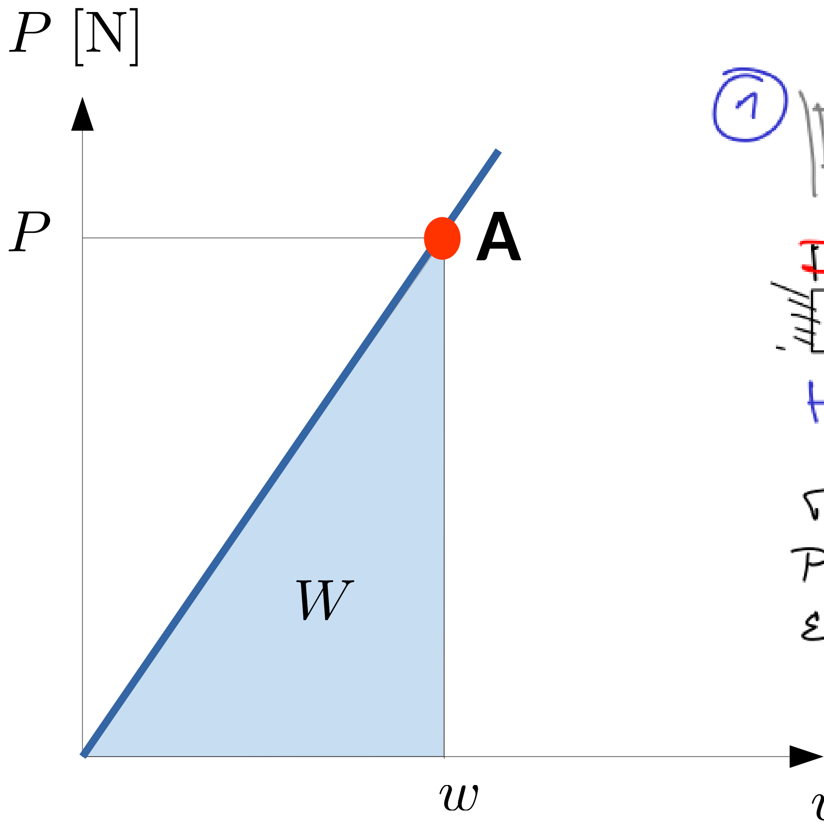

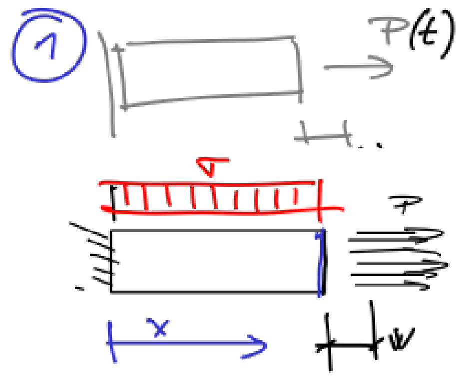

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Dissipated energy – elastic bar loaded in tension

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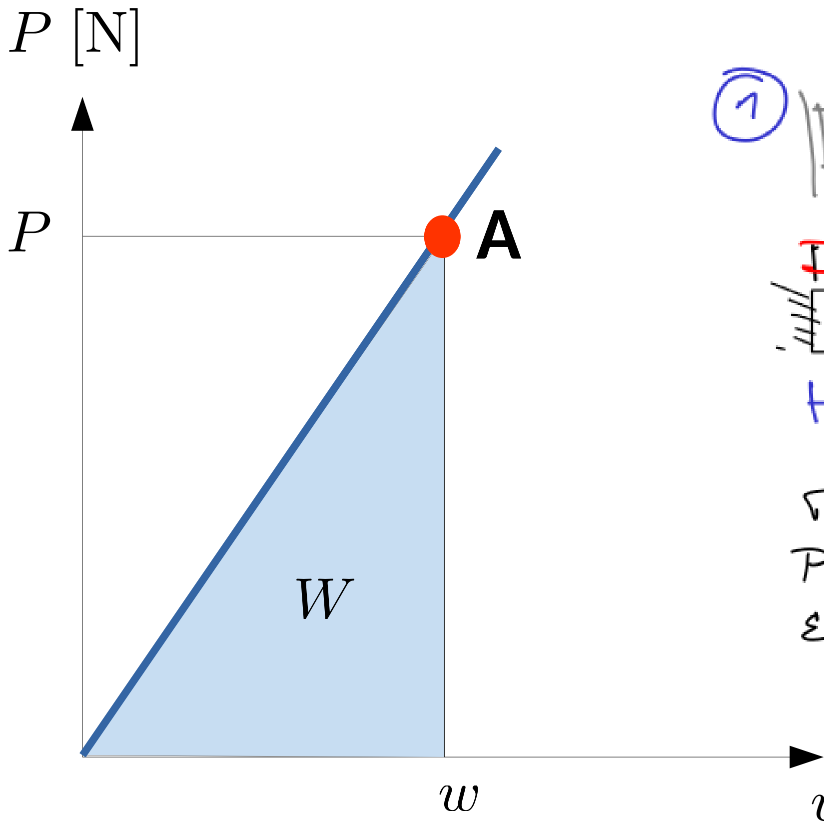

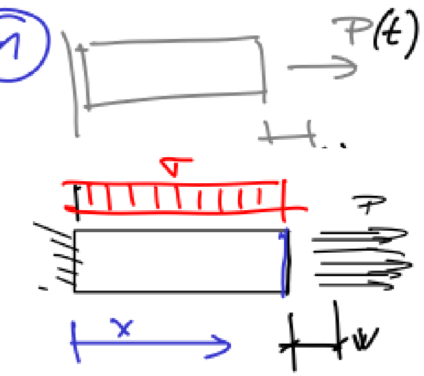

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$$U = \frac{1}{2} \cdot \frac{EA}{L} w \cdot w$$

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Dissipated energy – elastic bar loaded in tension

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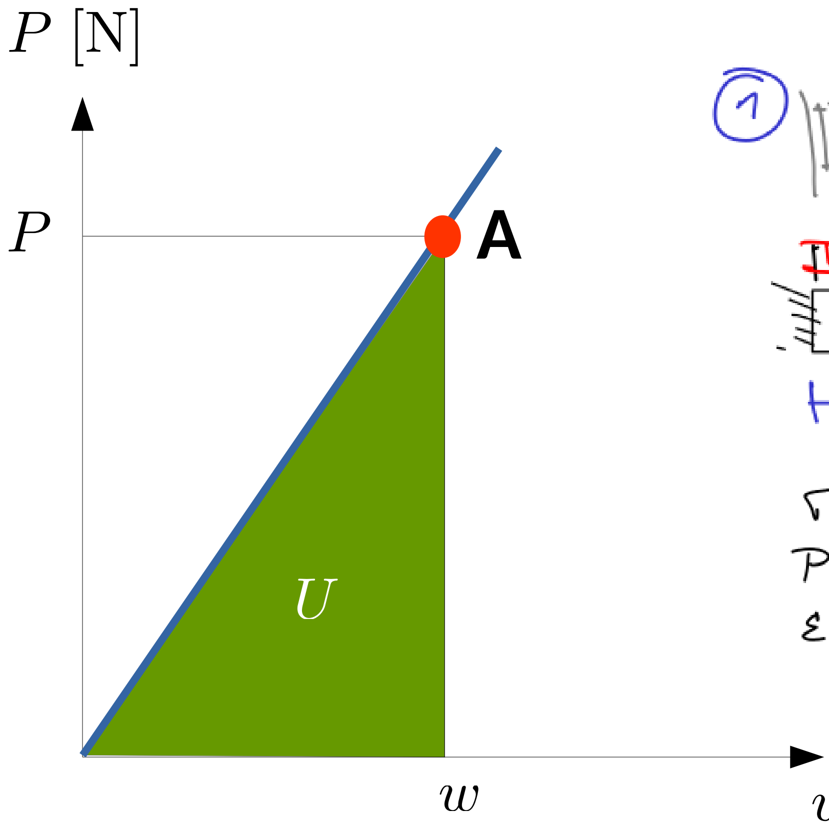

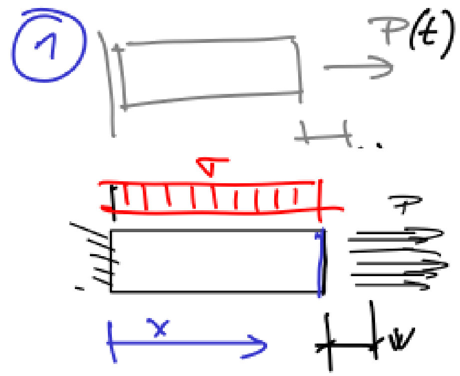

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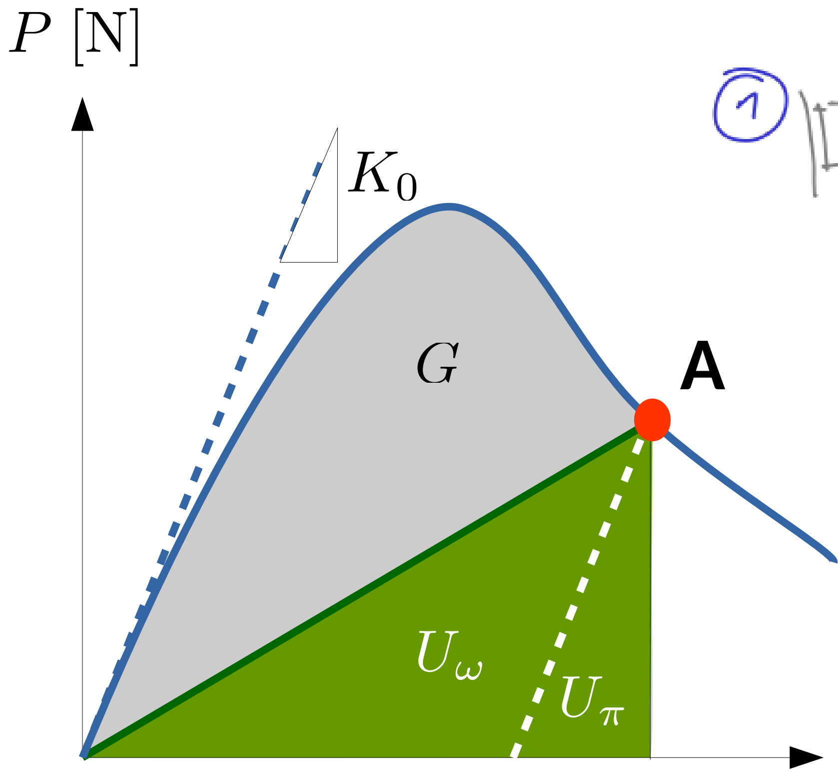
$$U = \frac{EA}{2L} \bar{w}^2$$

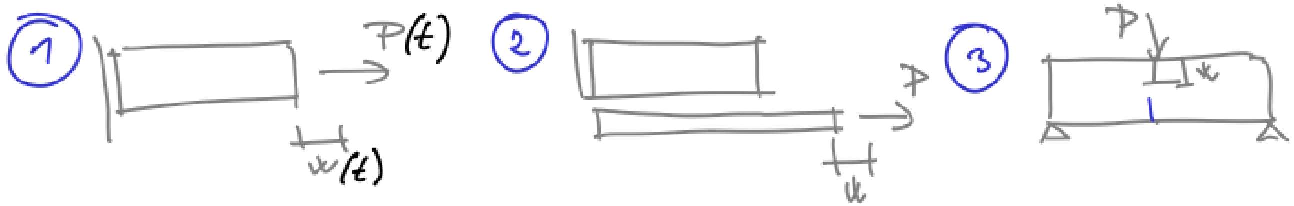
$$G = W - U = 0$$

NO DISSIPATION

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U$$

Dissipated energy – pull-out from rigid matrix



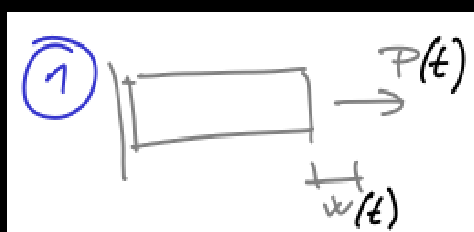


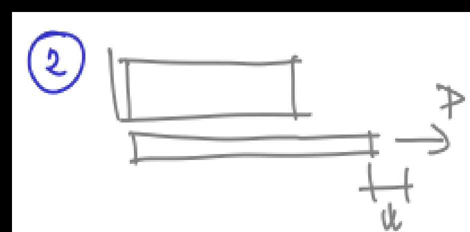
$$G = W - U = \int_0^w P(w)\,\mathrm{d}w - U(w)$$

material behavior, boundary conditions, geometry

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2}Pw \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U$$

elastic bar and frictional interface



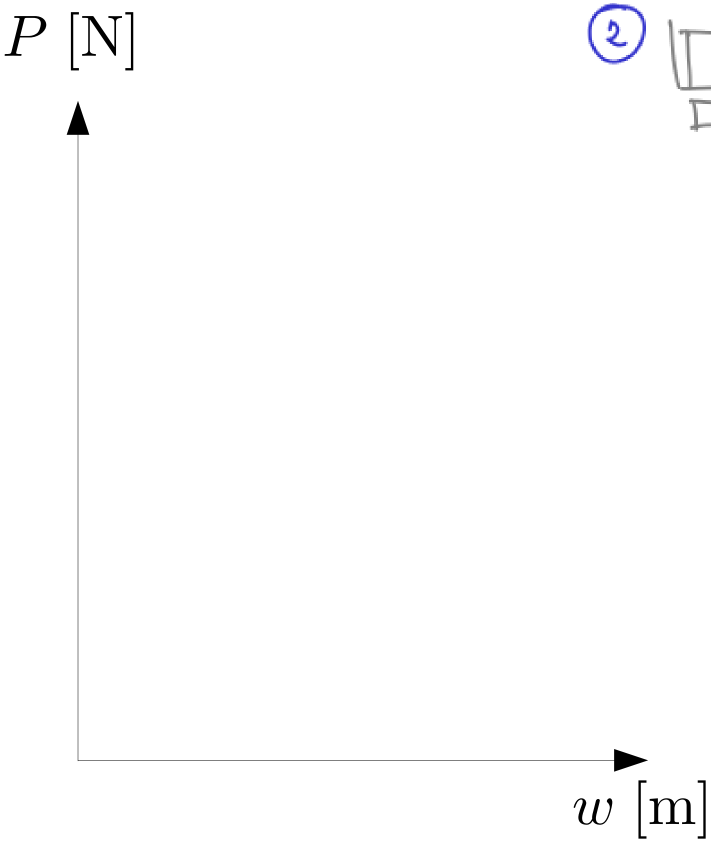


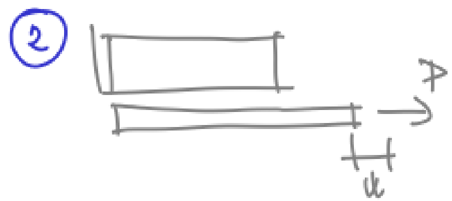
$G = 0$

$G = ?$

Dissipated energy – pull-out from rigid matrix

$$G = W - U = \int_0^w P(w)\,\mathrm{d}w - U(w)$$





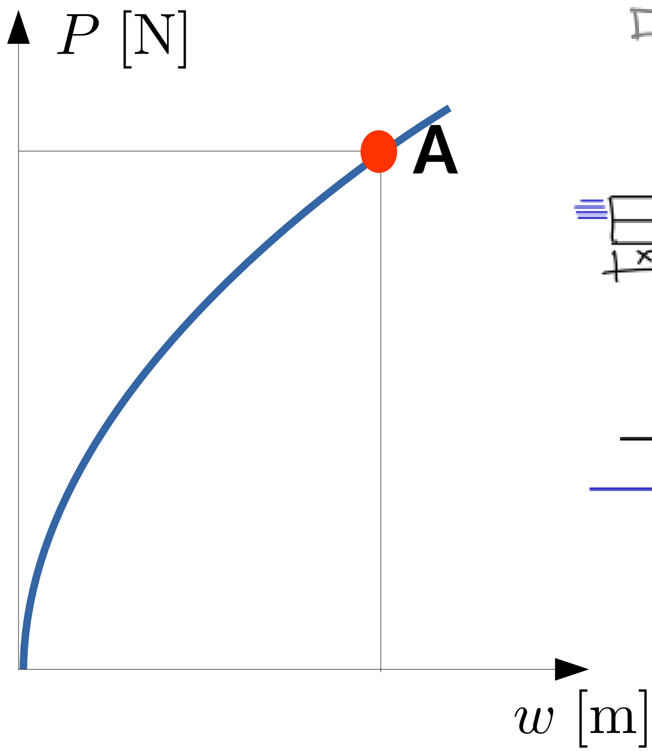
$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U$$

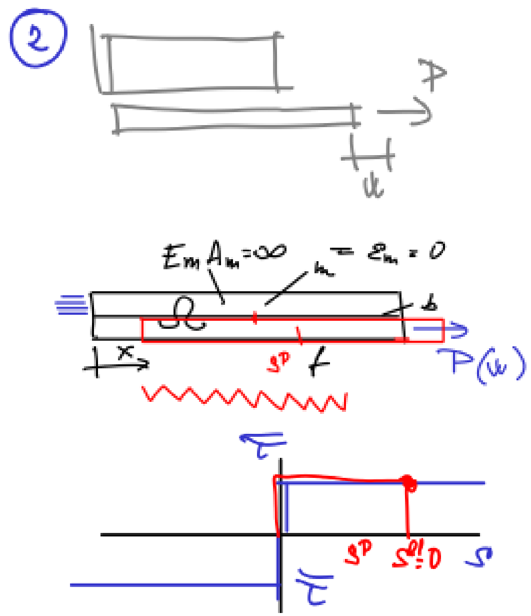
Dissipated energy – pull-out from rigid matrix

$$P = \sqrt{2} \cdot \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \sqrt{w}$$

[see Tour 2]

$$G = W - U = \int_0^w P(w)\,\mathrm{d}w - U(w)$$





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Dissipated energy – pull-out from rigid matrix

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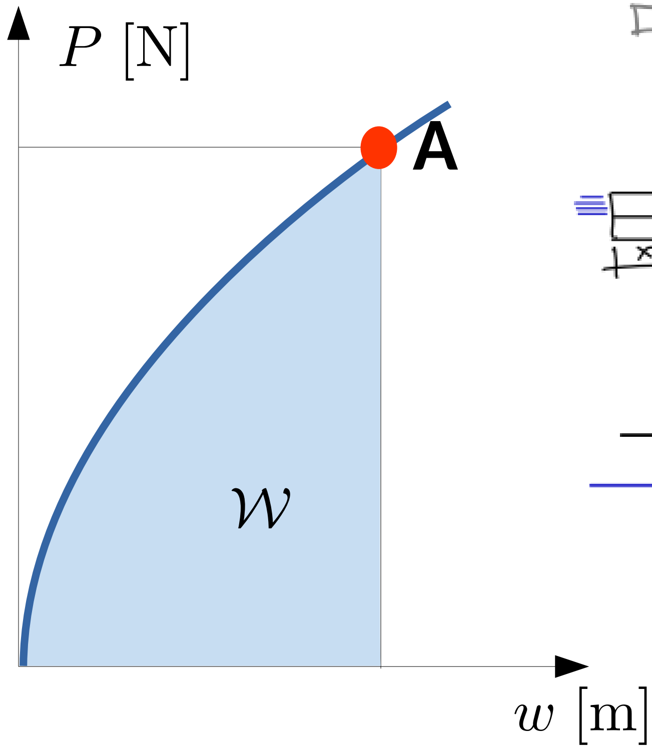

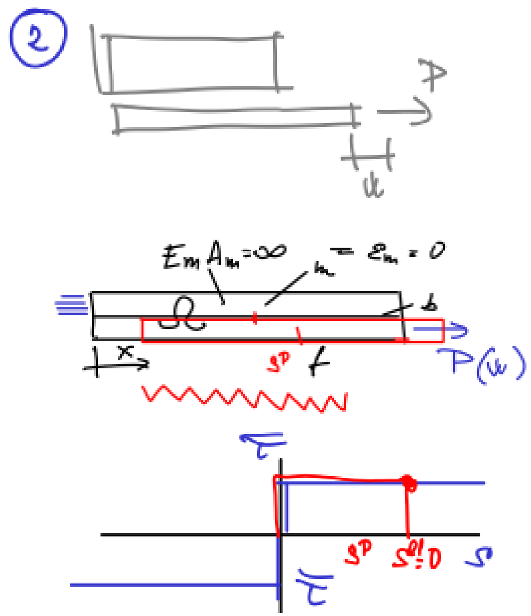

$$W = \frac{2\sqrt{2}}{3} \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \cdot w^{\frac{3}{2}}$$

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad W = \frac{2\sqrt{2}}{3} \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \cdot w^{\frac{3}{2}}$$

Dissipated energy – pull-out from rigid matrix

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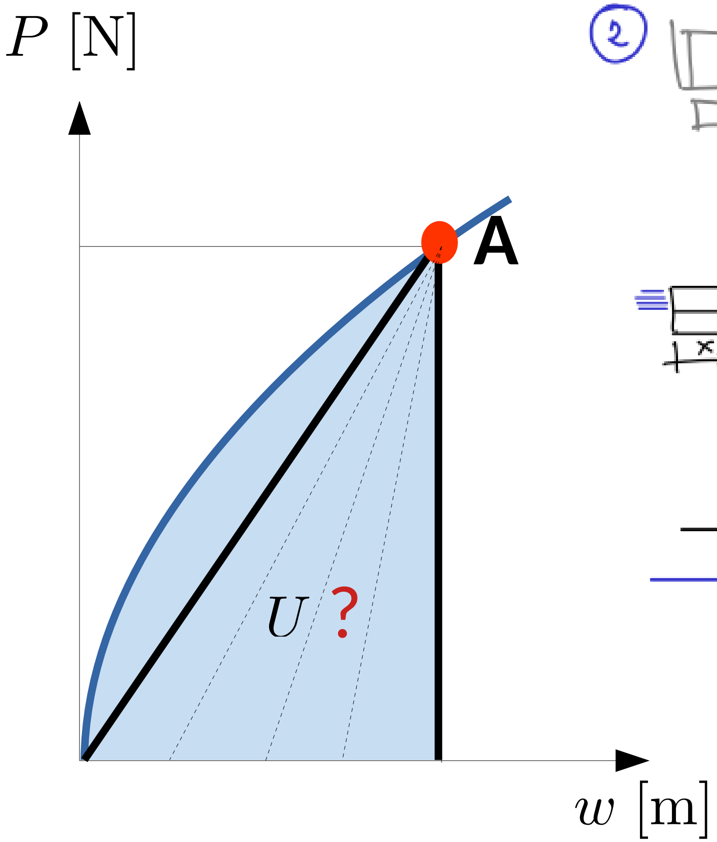
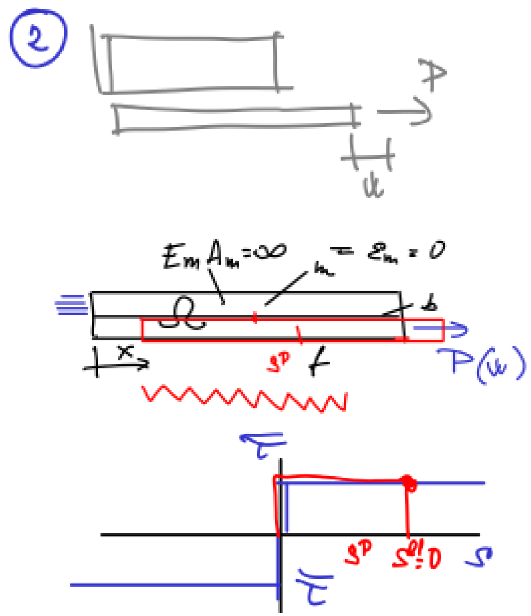
$$G = W - U = \int_0^w P(w)\,\mathrm{d}w - U(w)$$





$$U(w) = ?$$

How much energy was stored?

How would the pull-out test unload?

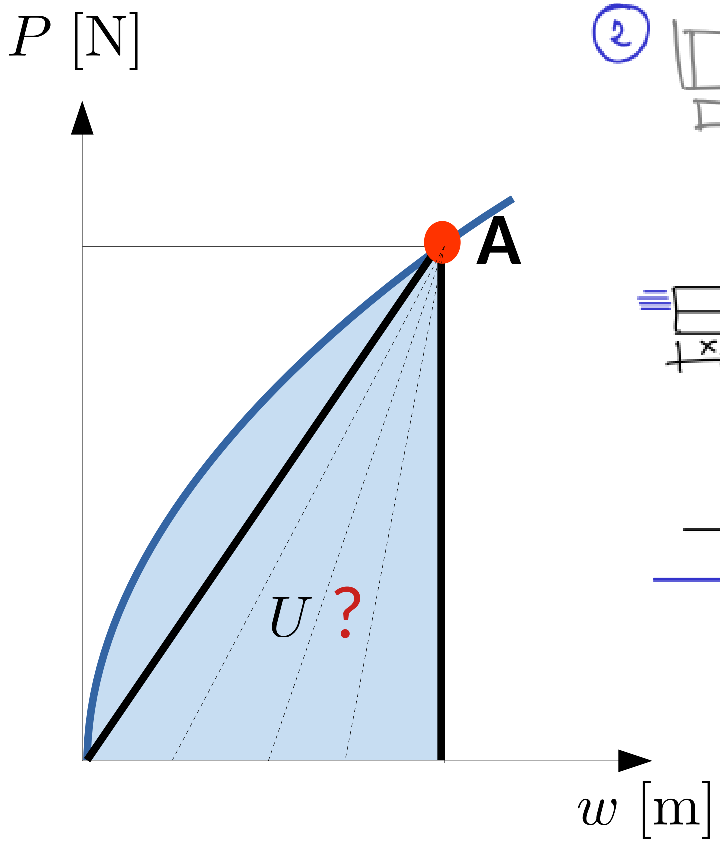
Maybe along the initial stiffness?

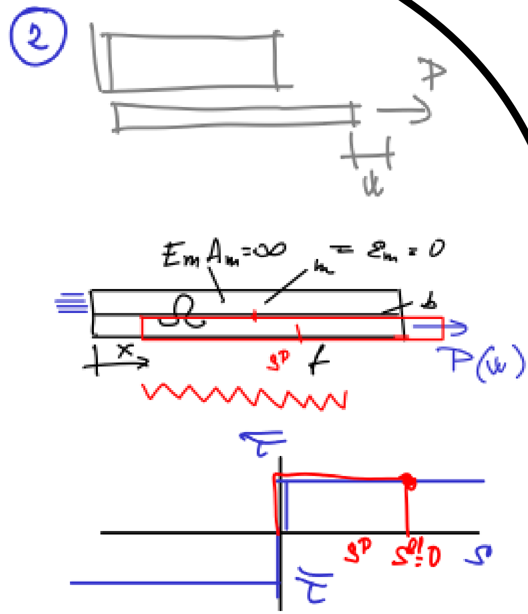
$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad W = \frac{2\sqrt{2}}{3}\sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \cdot w^{\frac{3}{2}}$$

Dissipated energy – pull-out from rigid matrix

$$P = \sqrt{2} \cdot \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \sqrt{w}$$

$$G = W - U = \int_0^w P(w)\,\mathrm{d}w - U(w)$$





$$U(w) = ?$$

How much energy was stored?

How would the pull-out test unload?

Maybe along the initial stiffness?

$$K_0 = \left.\frac{\partial P}{\partial w}\right|_{w=0} = \left.\frac{\sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}}}{2\sqrt{w}}\right|_0 = \infty$$

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Dissipated energy – pull-out from rigid matrix

$$P = \sqrt{2} \cdot \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \sqrt{w}$$

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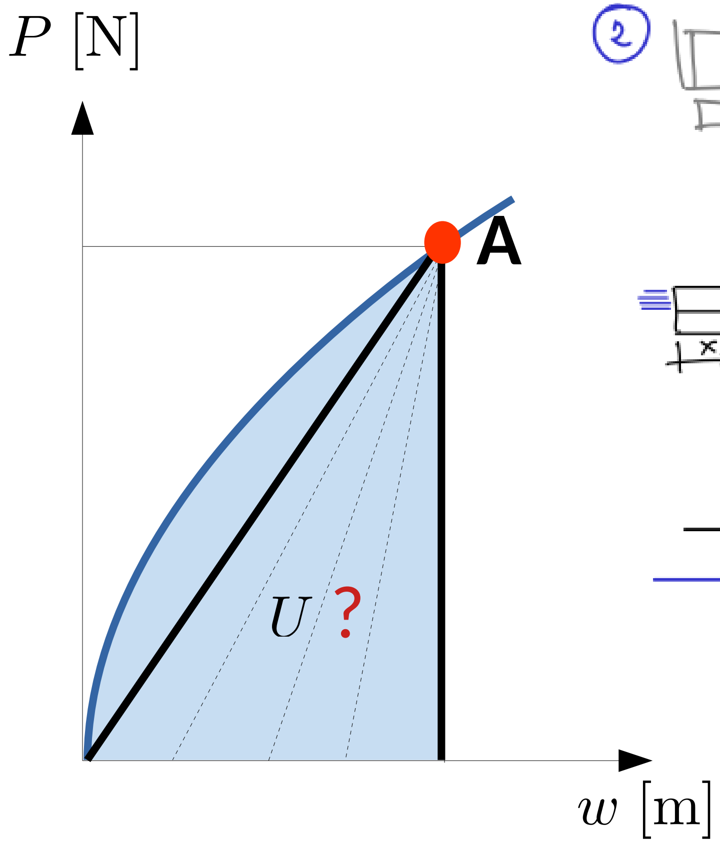

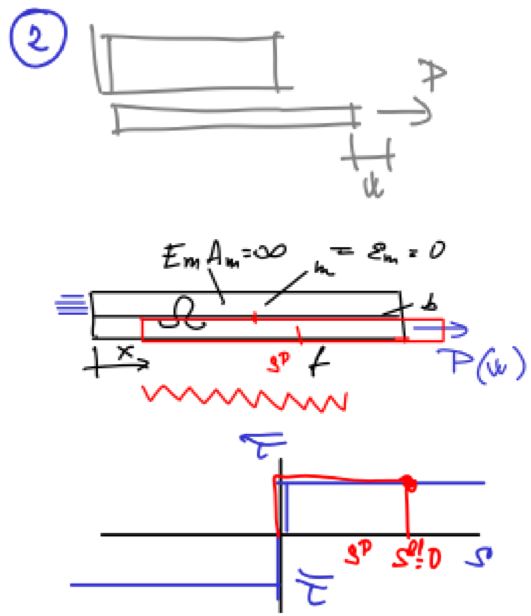

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$$K_0 = \left.\frac{\partial P}{\partial w}\right|_{w=0} = \left.\frac{\sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}}}{2\sqrt{w}}\right|_0 = \infty$$

But then - we would ignore the deformation of the pulled-out bar! How much energy does it store?

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad W = \frac{2\sqrt{2}}{3} \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \cdot w^{\frac{3}{2}}$$

Dissipated energy – pull-out from rigid matrix

$$P = \sqrt{2} \cdot \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \sqrt{w}$$

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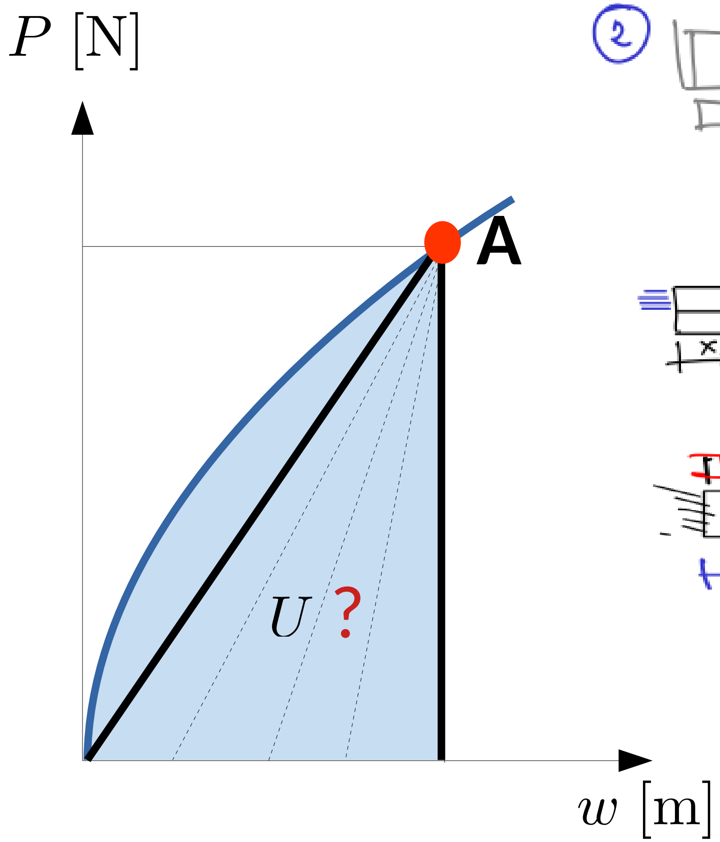

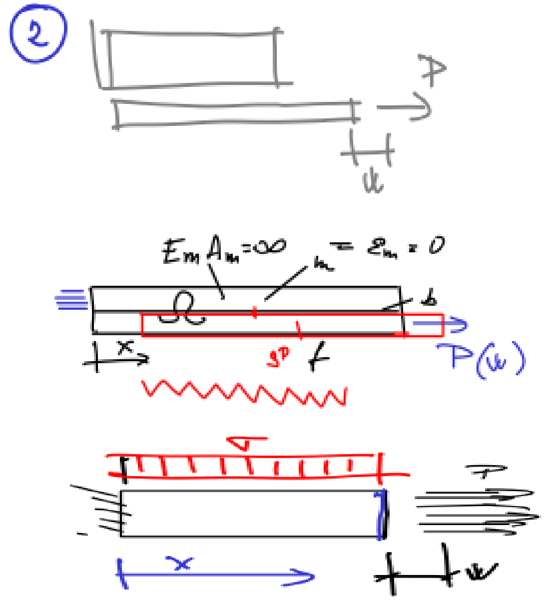

$$U(w) = ?$$

How much energy was stored?

How would the pull-out test unload?

Should we unload to the origin then?

Can we reuse the stored energy derived before for a tensile bar?

$$U = \frac{E_\mathrm{f} A_\mathrm{f}}{2L} w^2$$

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad W = \frac{2\sqrt{2}}{3}\sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \cdot w^{\frac{3}{2}}$$

Dissipated energy – pull-out from rigid matrix

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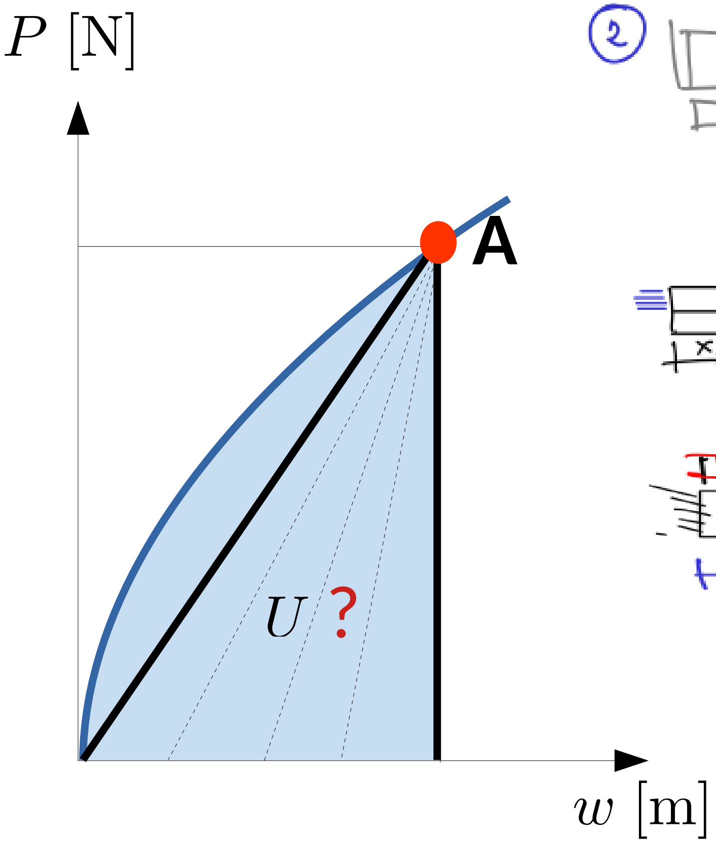

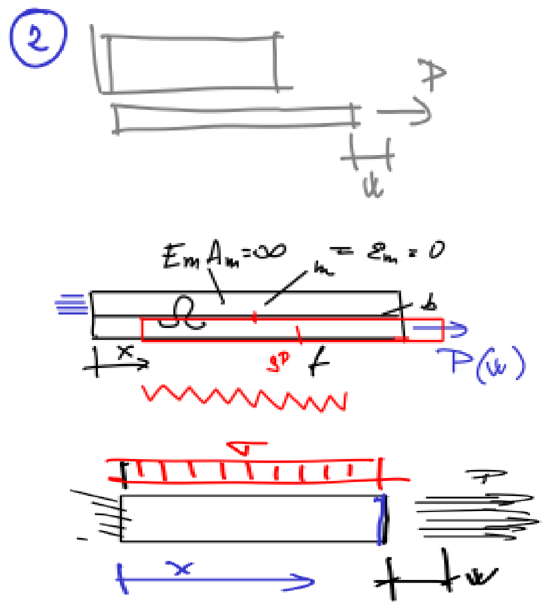

$\sigma = E \varepsilon$ - const. law

$\varepsilon = \frac{w}{L}$ - kinematics

$$U(w) = ?$$

How much energy was stored?
How would the pull-out test unload?
Should we unload to the origin then?

Can we reuse the stored energy derived before for a tensile bar?

$$U = \frac{E_\mathrm{f} A_\mathrm{f}}{2L} w^2$$

NO – as it considered constant strain and stress along the bar

$$W = \int_0^w P(w) \, \mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad W = \frac{2\sqrt{2}}{3} \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \cdot w^{\frac{3}{2}}$$

Dissipated energy – pull-out from rigid matrix

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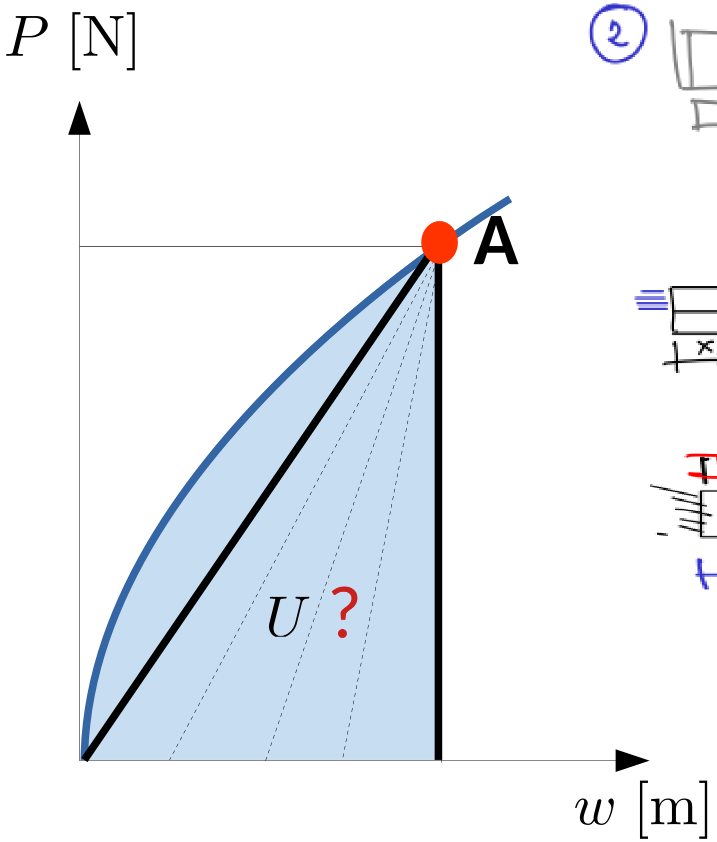

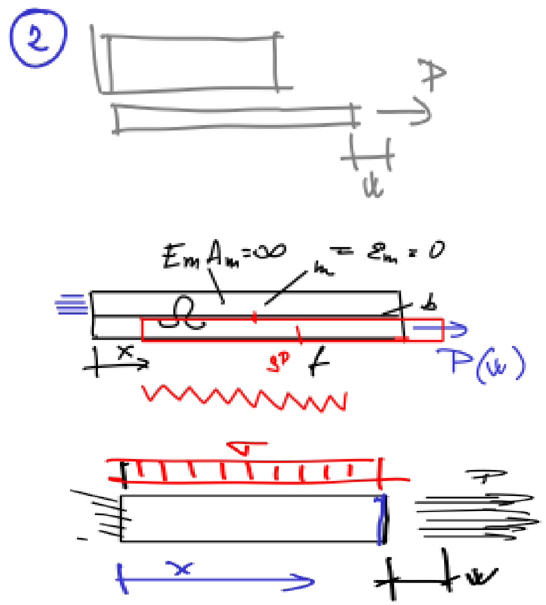

$\sigma = E\varepsilon$ - const. law

$\varepsilon = \frac{w}{L}$ - kinematics

$$U(w) = ?$$

How much energy was stored?
How would the pull-out test unload?

Solution: In case of elastic bar we actually evaluated an integral over energy stored in material points

$$U = \int_\Omega \sigma_\mathrm{el}(x) \varepsilon_\mathrm{el}(x)\,\mathrm{d}x$$

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad W = \frac{2\sqrt{2}}{3} \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \cdot w^{\frac{3}{2}}$$

Dissipated energy – pull-out from rigid matrix

$$P = \sqrt{2} \cdot \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \sqrt{w}$$

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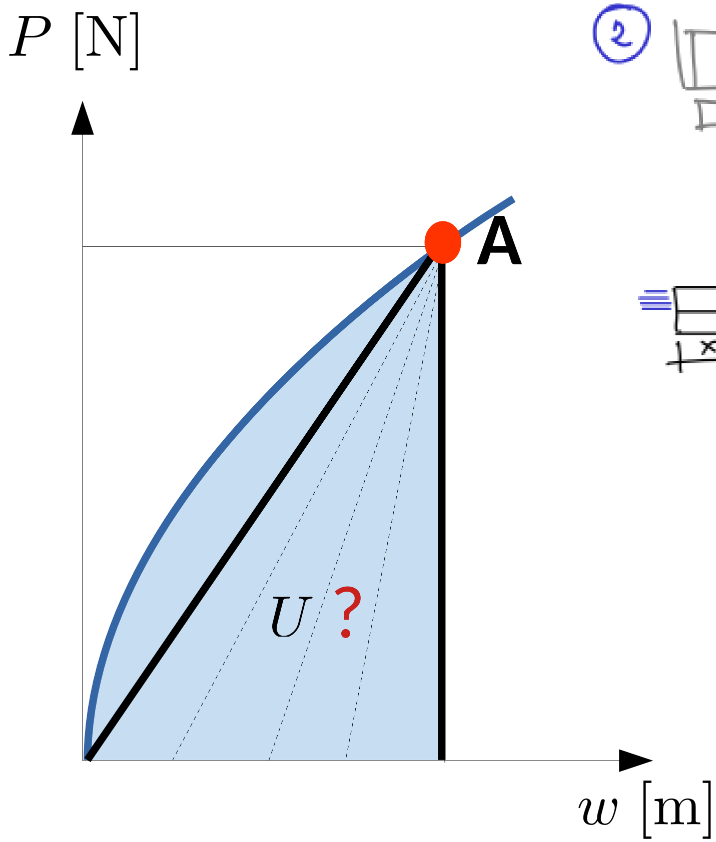

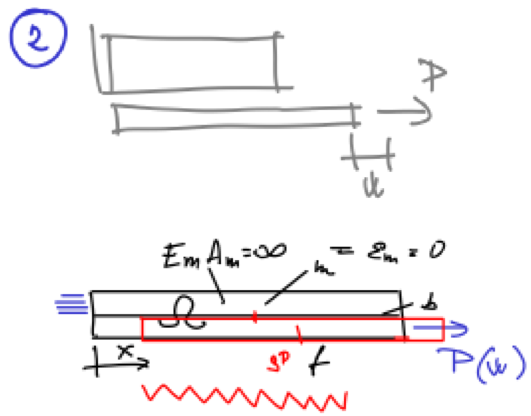

$$U(w) = \int_\Omega \sigma_\mathrm{el}(x) \varepsilon_\mathrm{el}(x)\,\mathrm{d}x$$

We already know these two fields for any value of control pull-out displacement w

They were derived in Tour 2:

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad W = \frac{2\sqrt{2}}{3} \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \cdot w^{\frac{3}{2}}$$

Dissipated energy – pull-out from rigid matrix

$$P = \sqrt{2} \cdot \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \sqrt{w}$$

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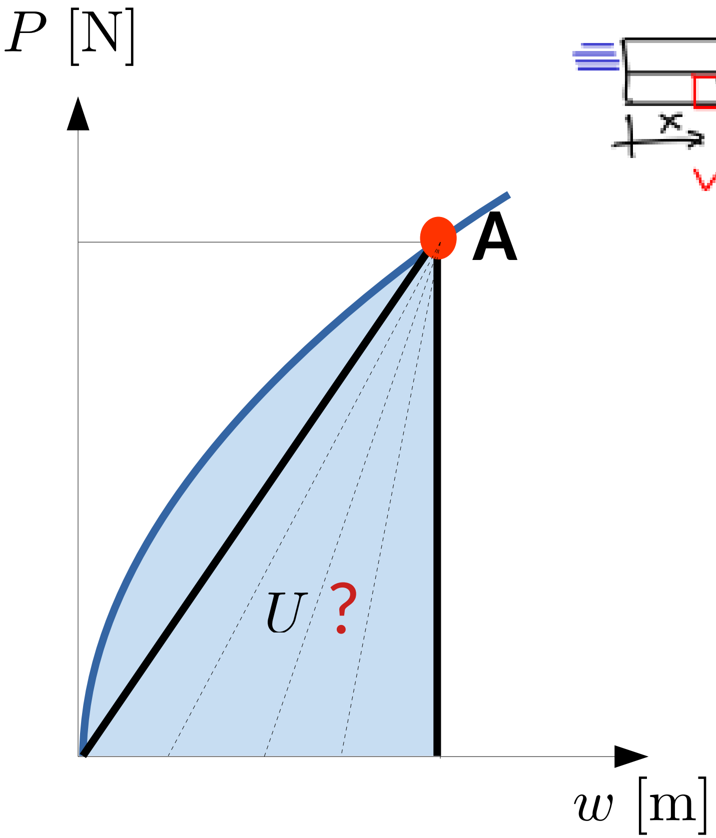

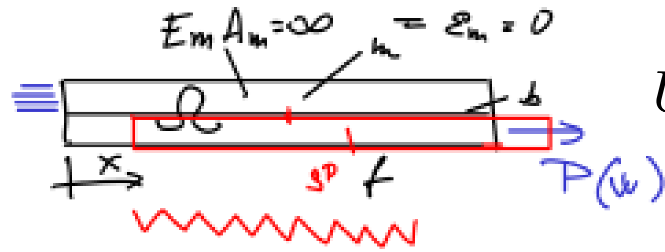

$$U(w) = \int_\Omega \sigma_\mathrm{el}(x) \varepsilon_\mathrm{el}(x)\,\mathrm{d}x$$

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Dissipated energy – pull-out from rigid matrix

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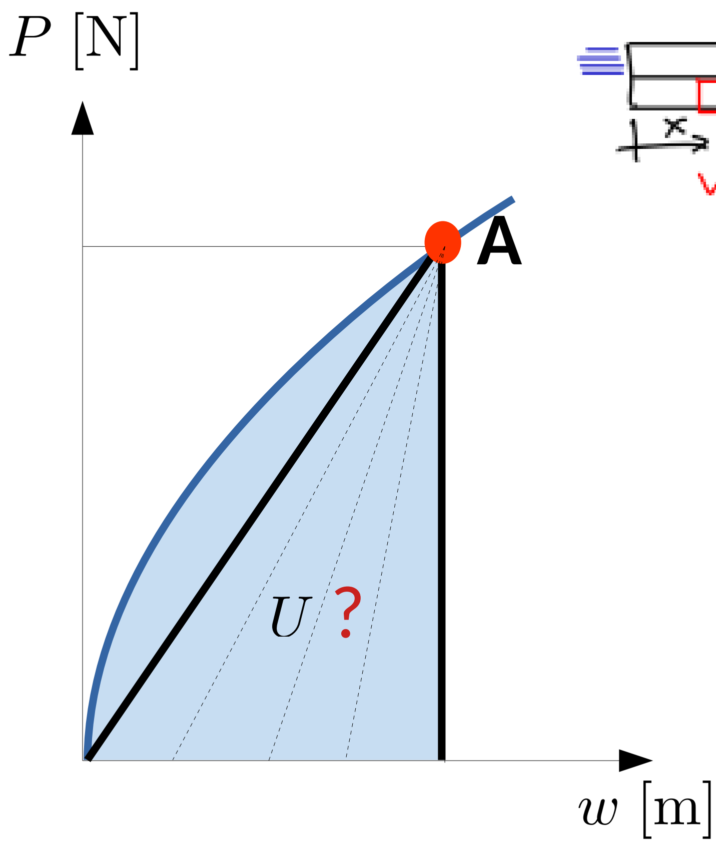

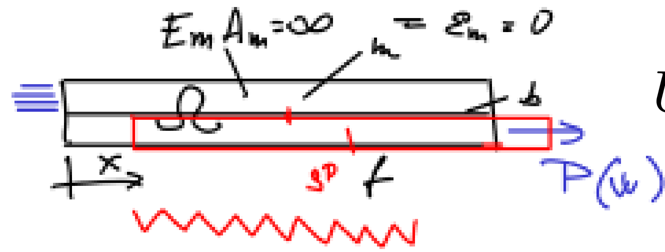

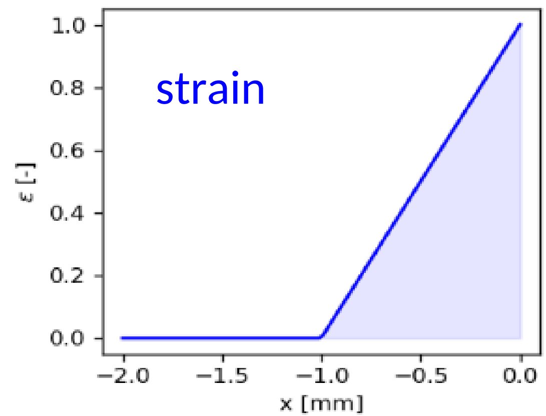

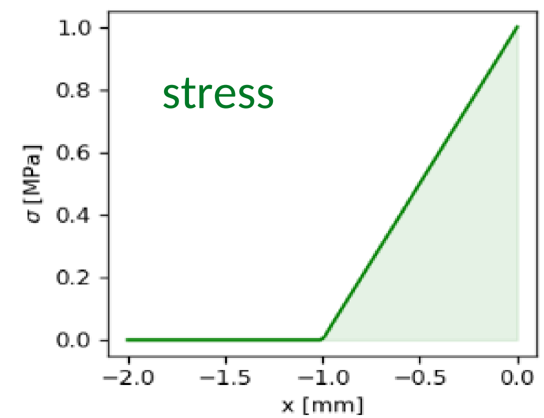

[see Tour 2]

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Dissipated energy – pull-out from rigid matrix

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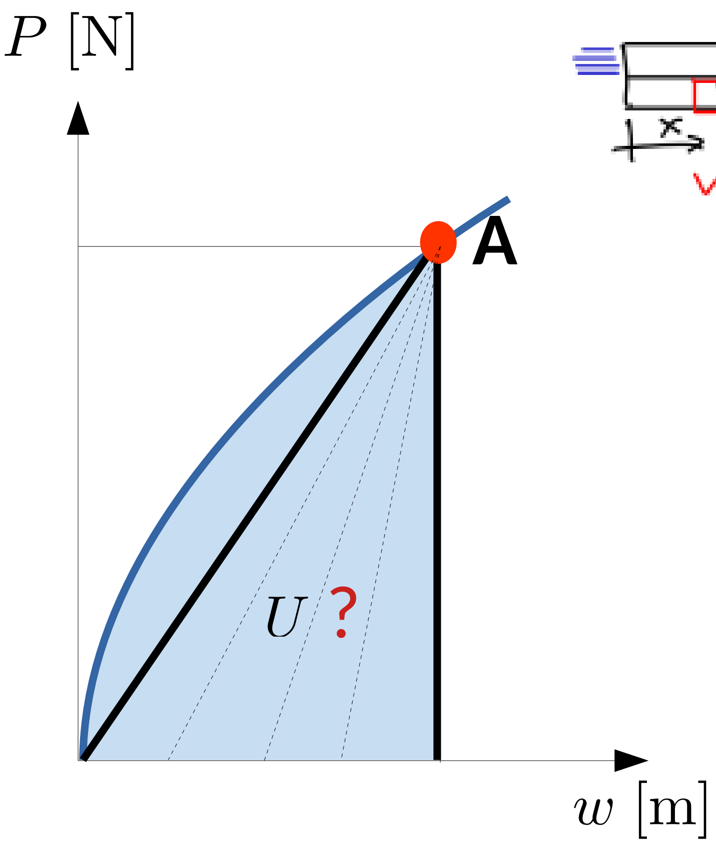

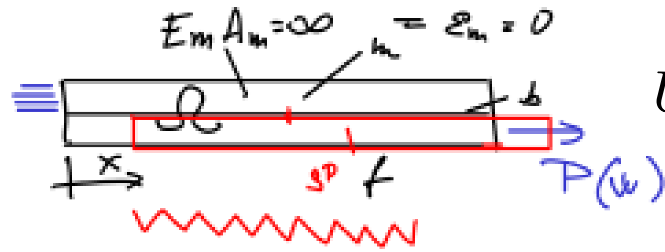

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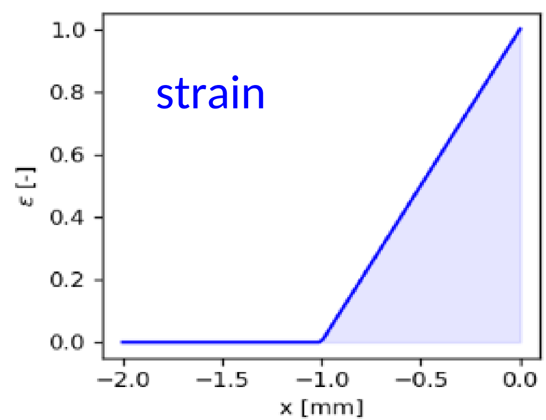

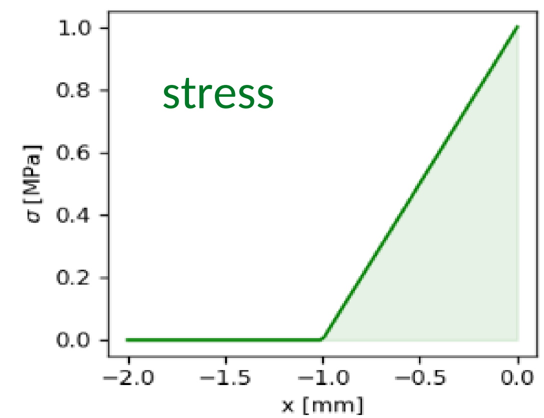

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Dissipated energy – pull-out from rigid matrix

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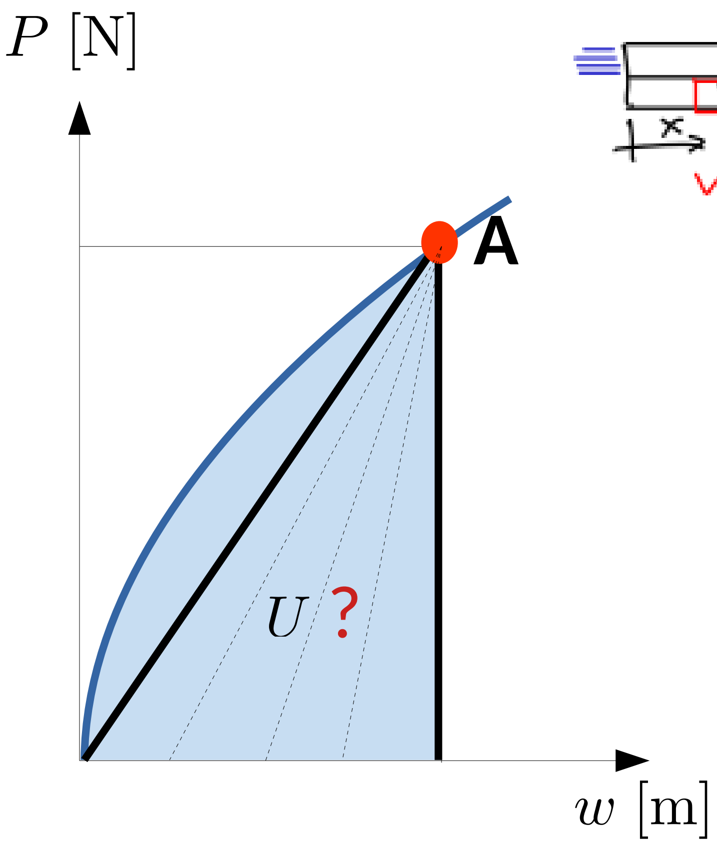

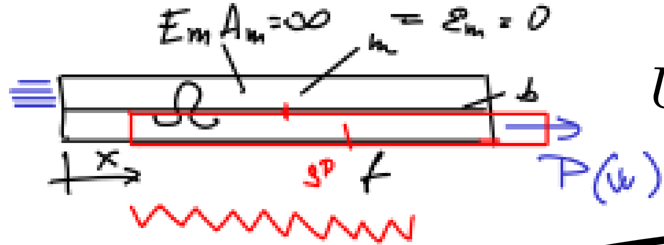

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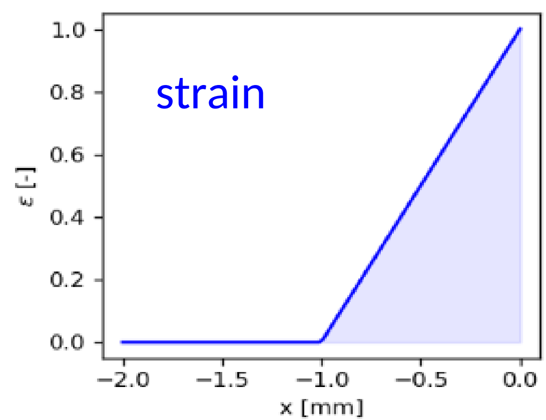

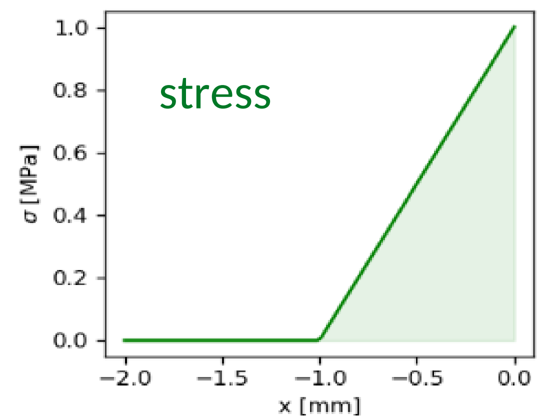

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Dissipated energy – pull-out from rigid matrix

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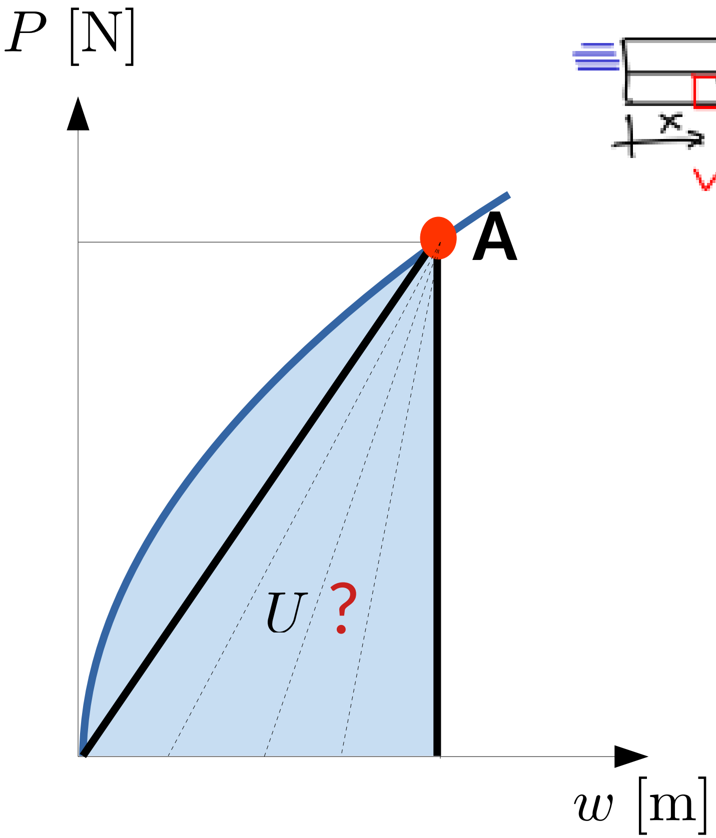

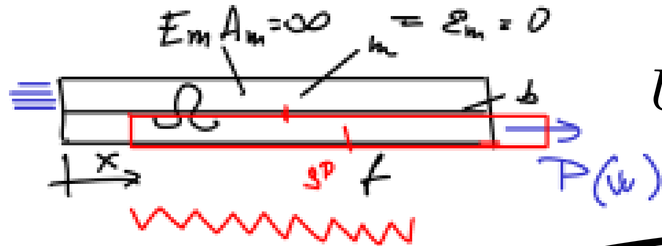

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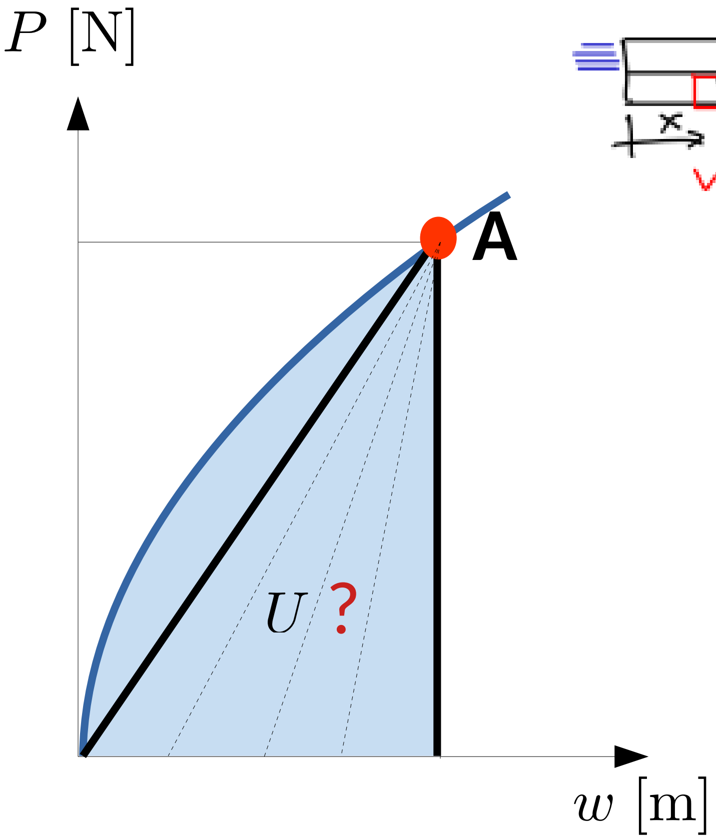

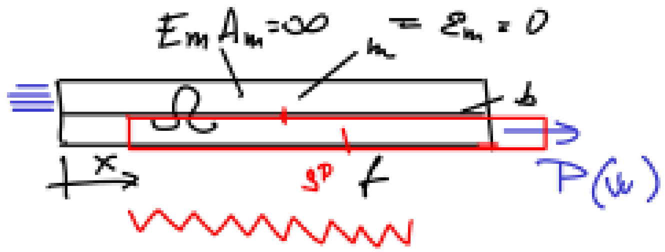

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Dissipated energy – pull-out from rigid matrix

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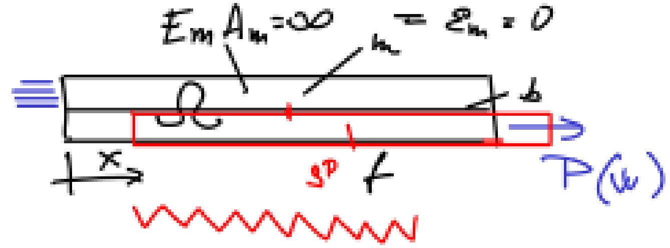

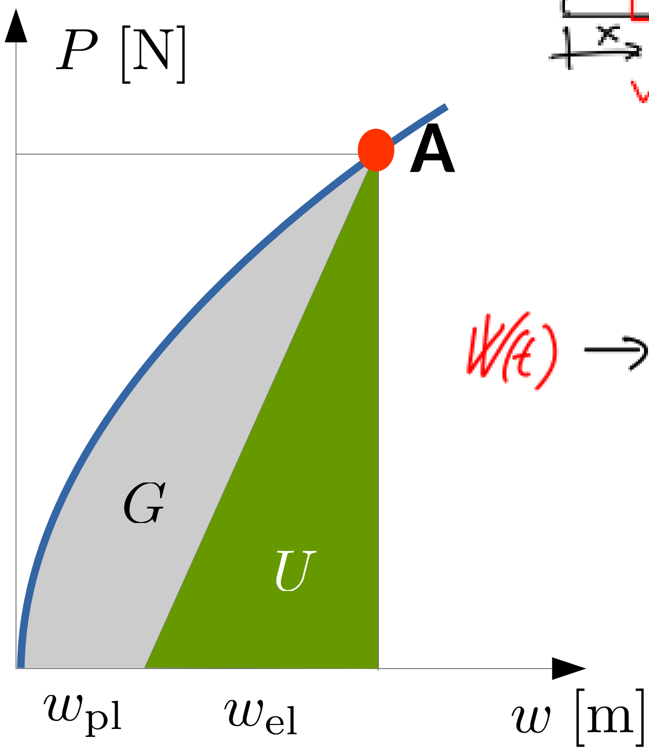

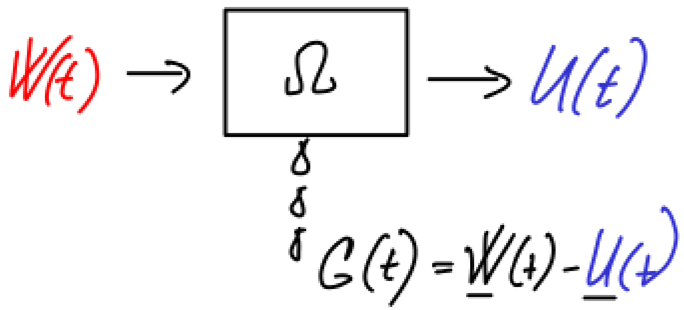

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DISSIPATED = STORED?

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Dissipated energy – pull-out from rigid matrix

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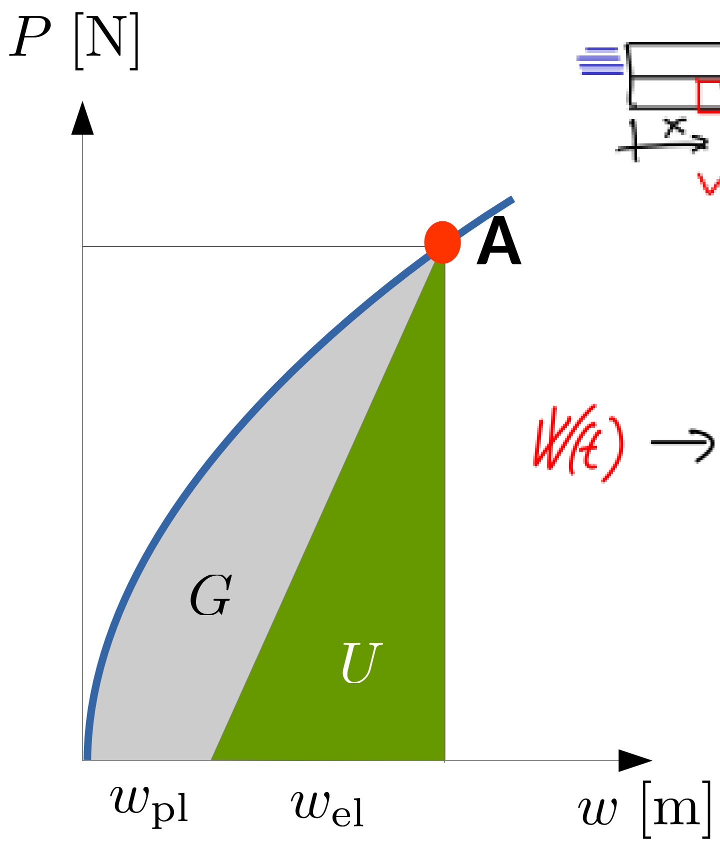

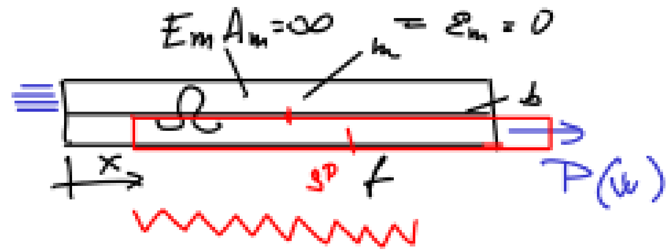

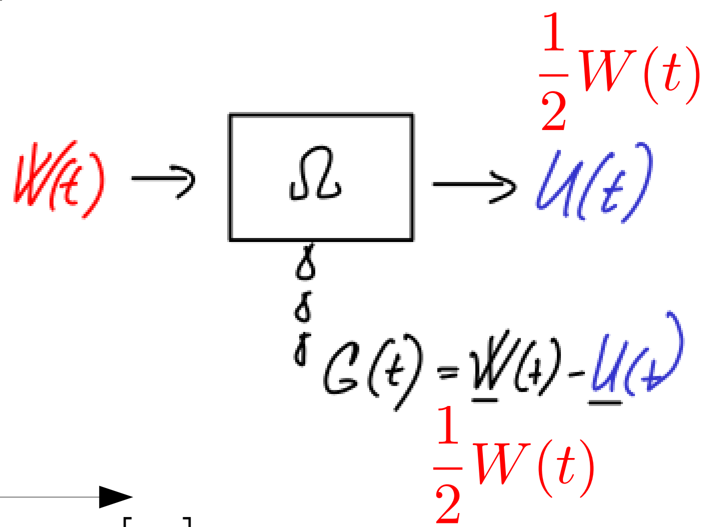

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Dissipated energy – pull-out from rigid matrix

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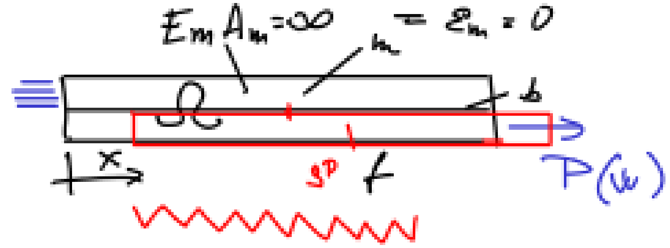

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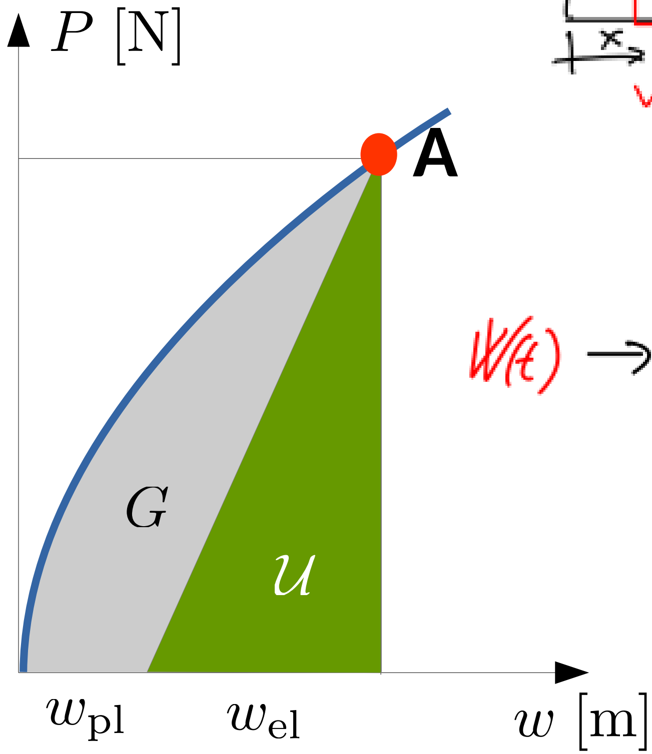
$$\implies G = U$$



$$W(t) \rightarrow \boxed{\Omega} \rightarrow \overset{\frac{1}{2}W(t)}{U(t)}$$

$$G(t) = \underline{W}(t) - \underbrace{\underline{U}(t)}_{\frac{1}{2}W(t)}$$

1. How much slip remains after unloading?

2. Dissipation and storage processes spatially separable?

$$W = \int_0^w P(w)\,\mathrm{d}w \qquad U_\omega = \frac{1}{2}Pw \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad U = \int_{\Omega_\mathrm{el}} \sigma(x)\varepsilon(x)\,\mathrm{d}x$$

Dissipated energy – pull-out from rigid matrix

$$P = \sqrt{2} \cdot \sqrt{E_\mathrm{f} A_\mathrm{f} p \bar{\tau}} \sqrt{w}$$

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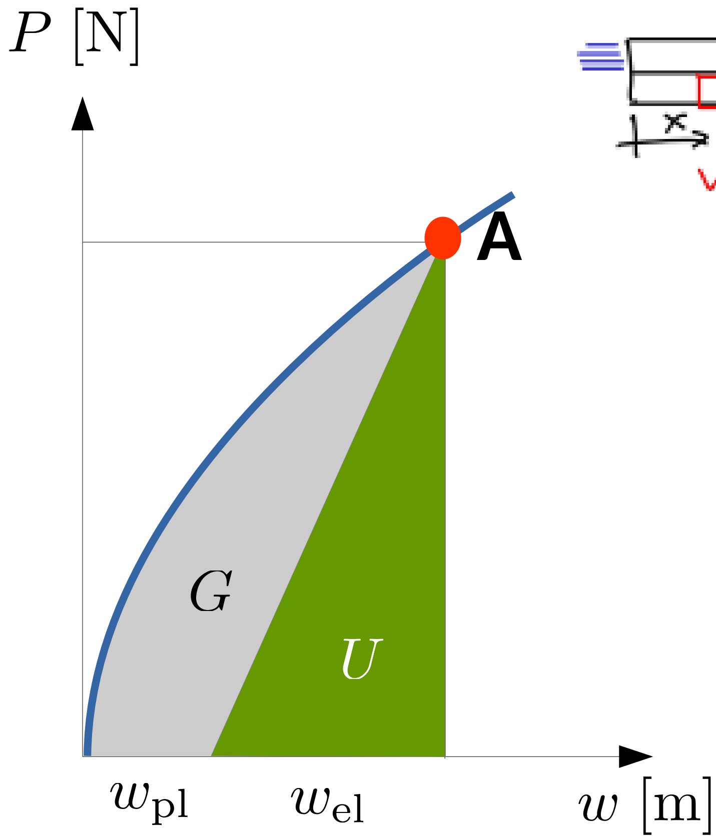

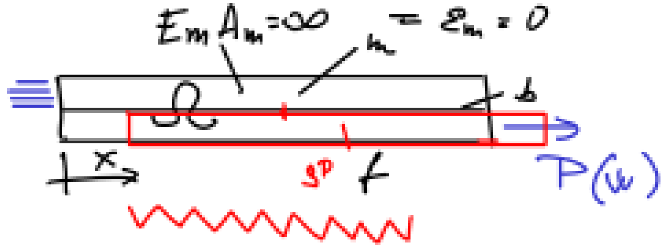

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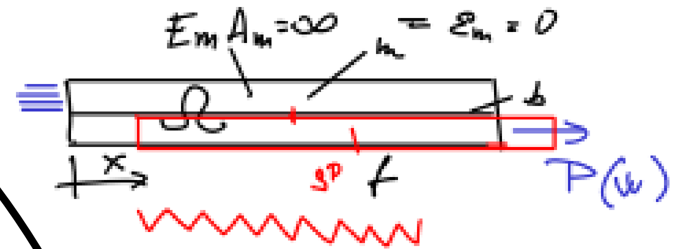

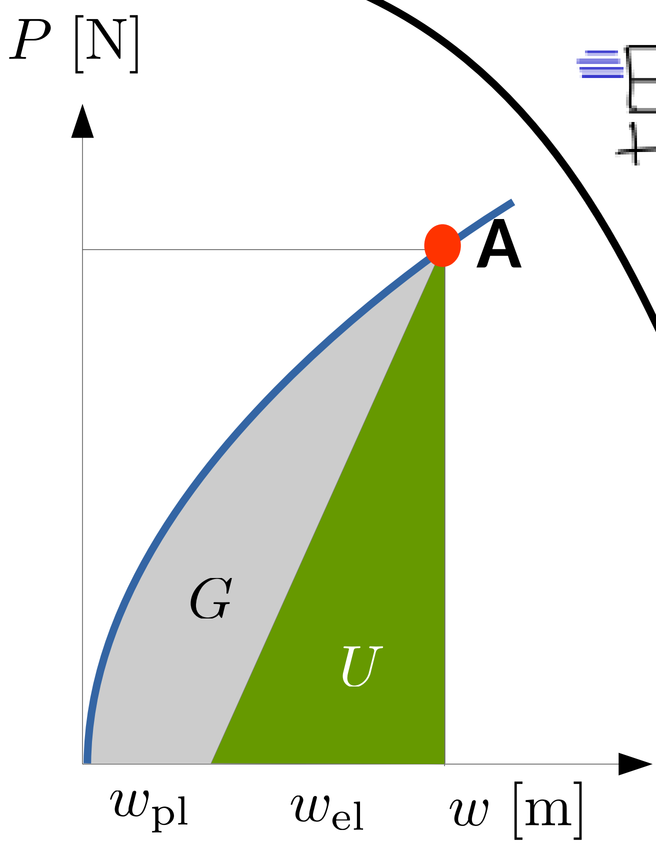

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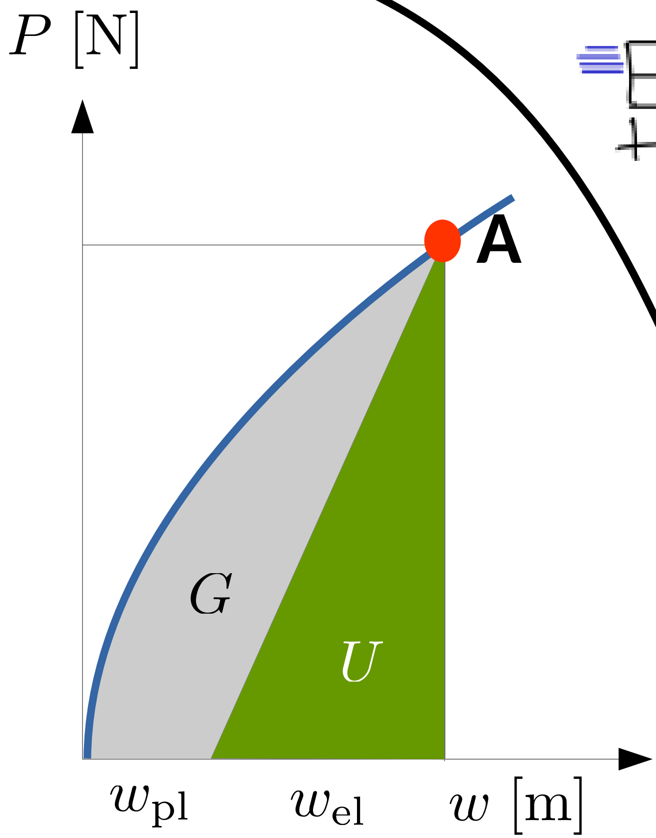

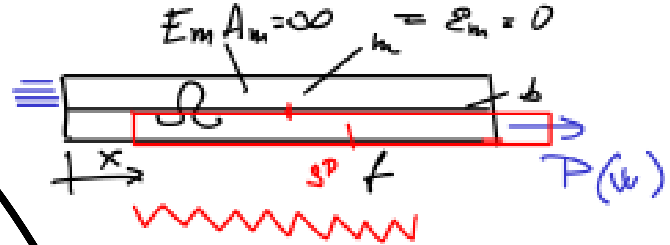

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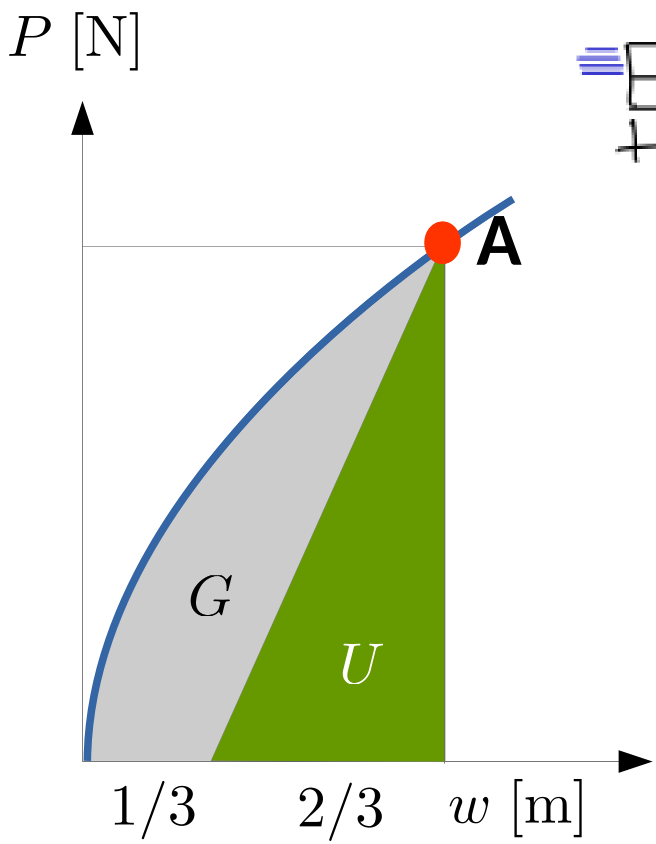

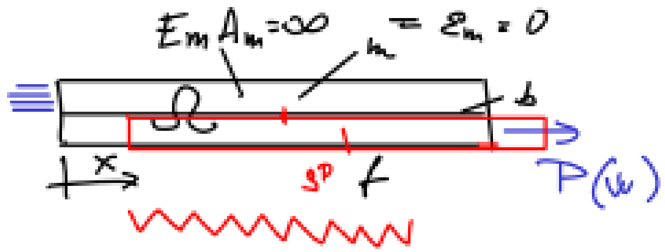

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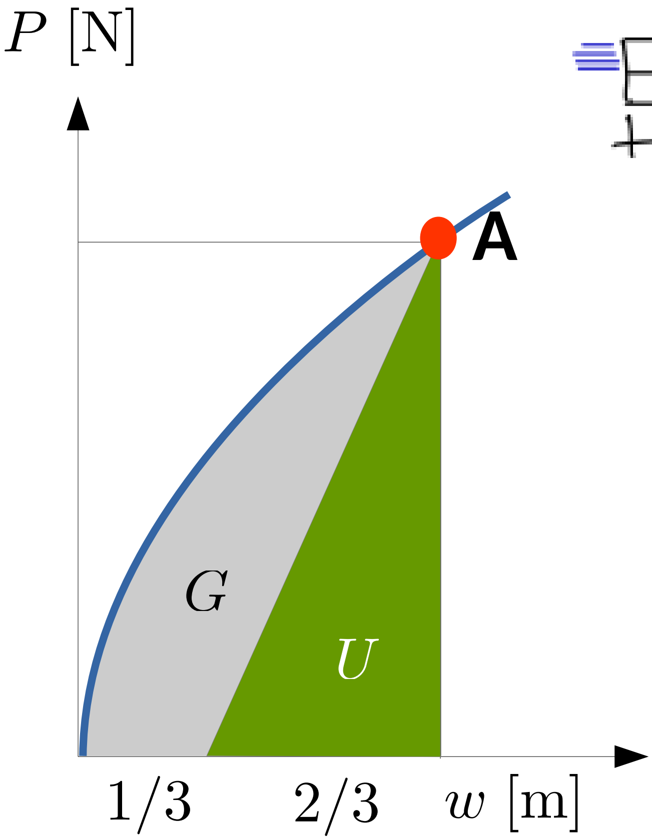

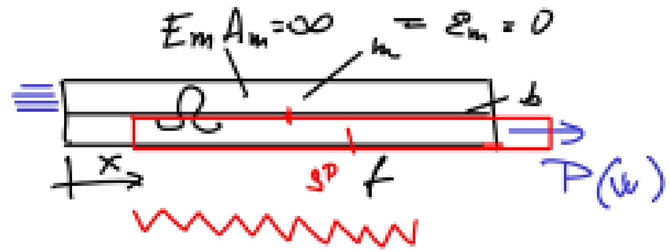

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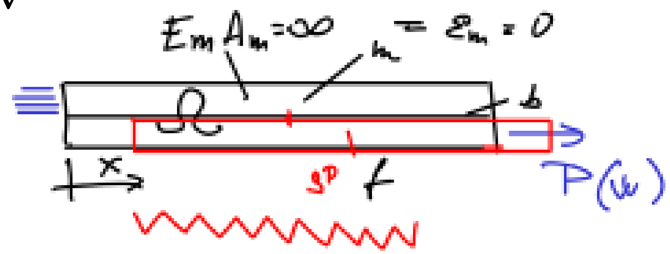

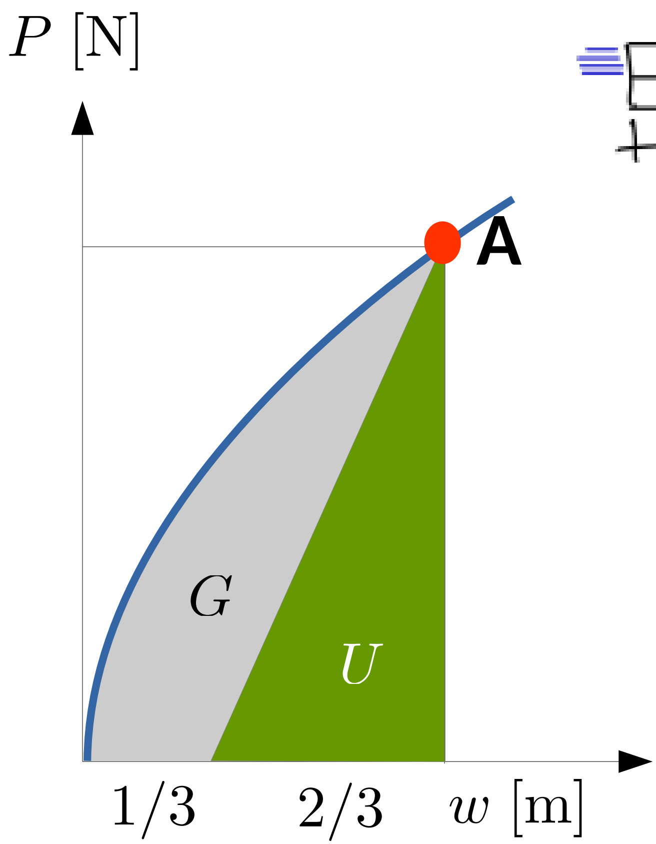

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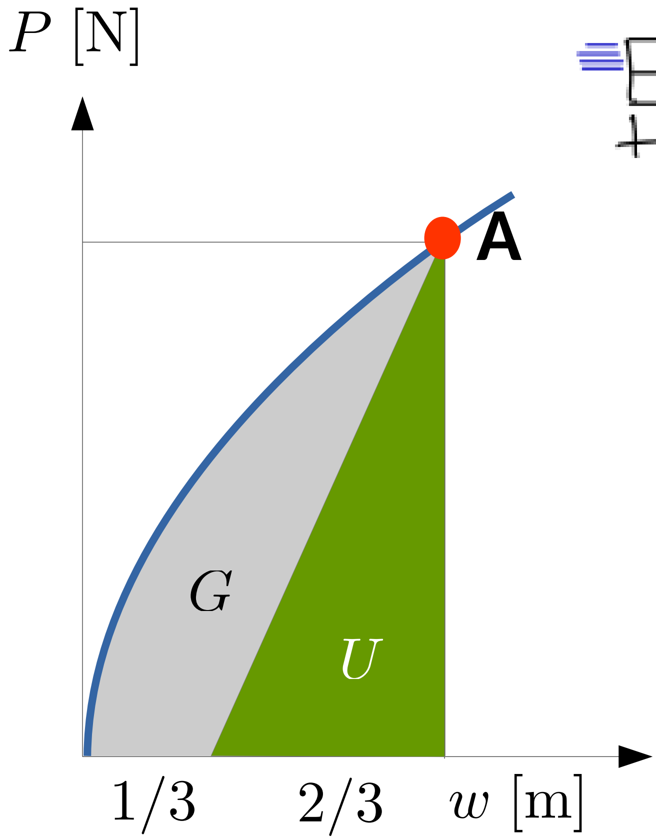

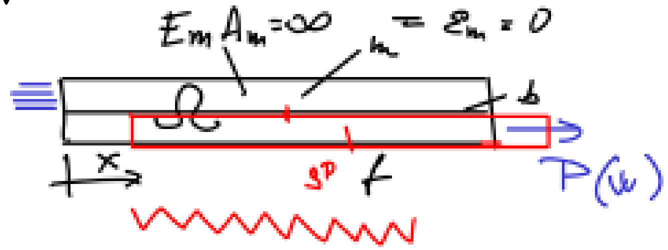

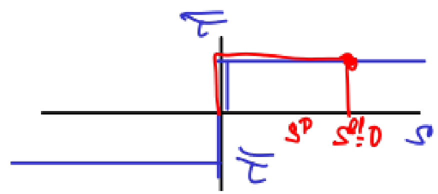

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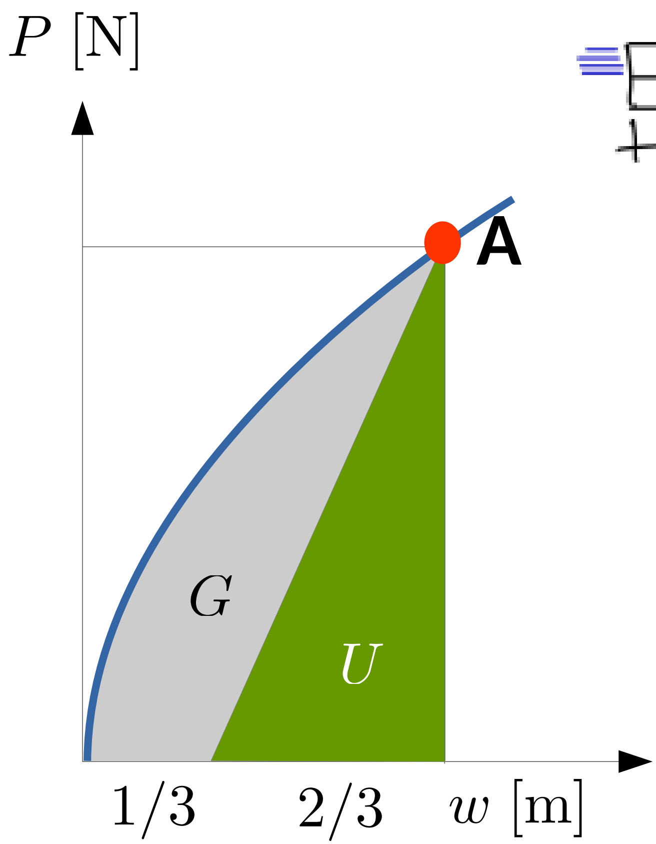

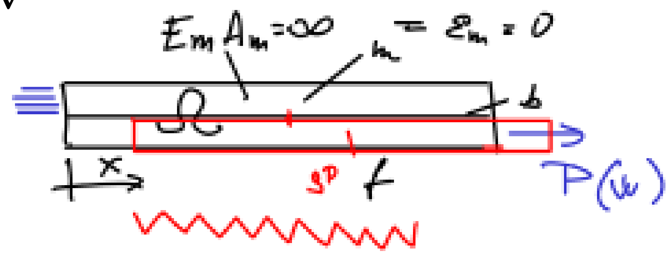

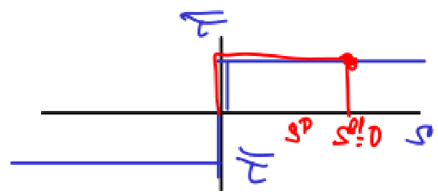

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What? No damage, Perfect plasticity, but unloading somewhere in-between?

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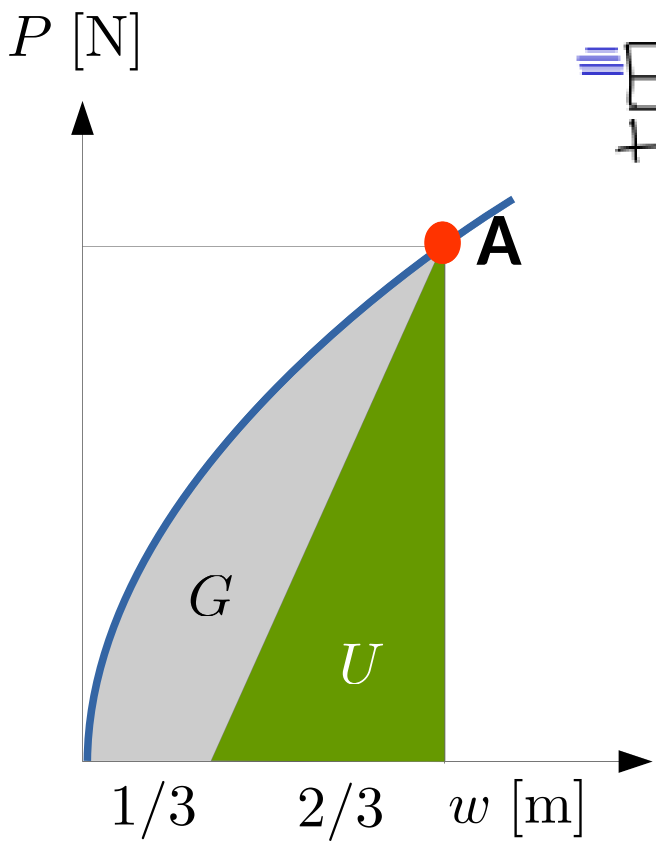

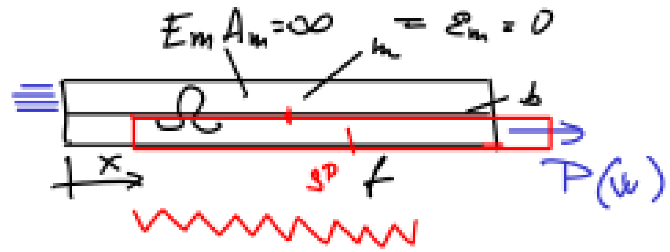

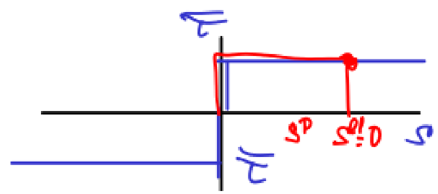

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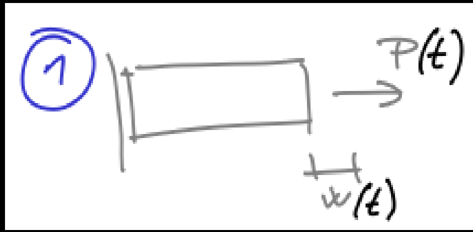
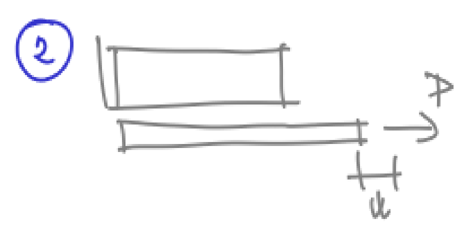
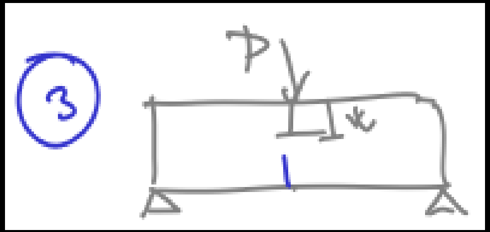
separation of domains with energy storage and dissipation







$G = 0$

$G = U$

$G = ?$

Dissipated energy → it tends to localize into a small material volume

Realize:

The example combined two energetic devices
one fully dissipative device → no energy storage (ideally plastic bond), and
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Idea:

If it is possible to integrate the stored energy in all material points over the domain can we also integrate the released energy over the domain?

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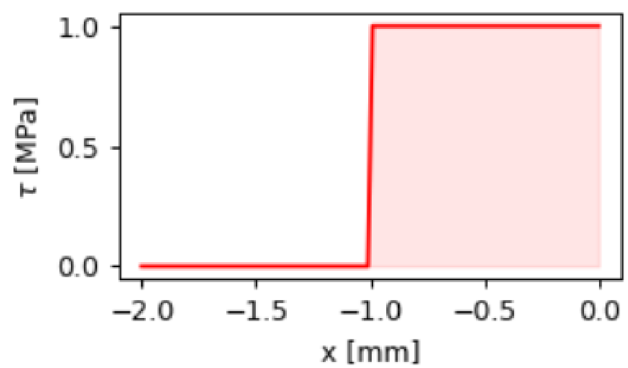

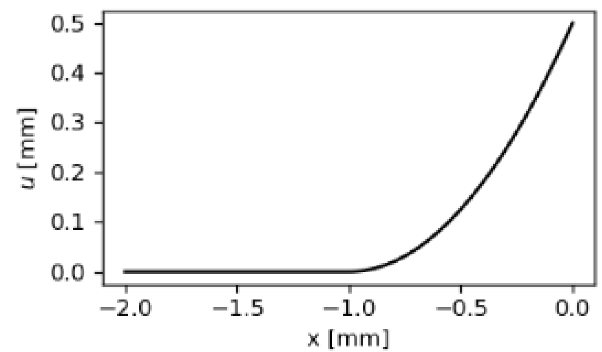

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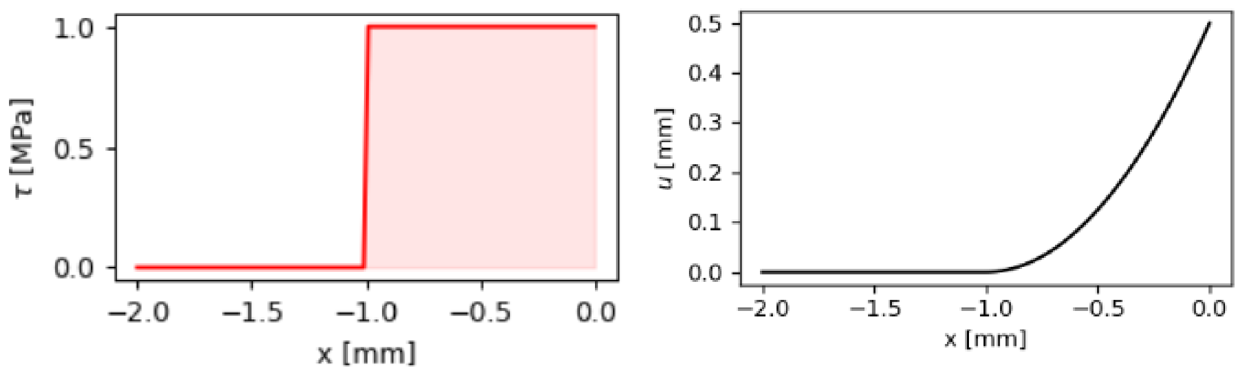

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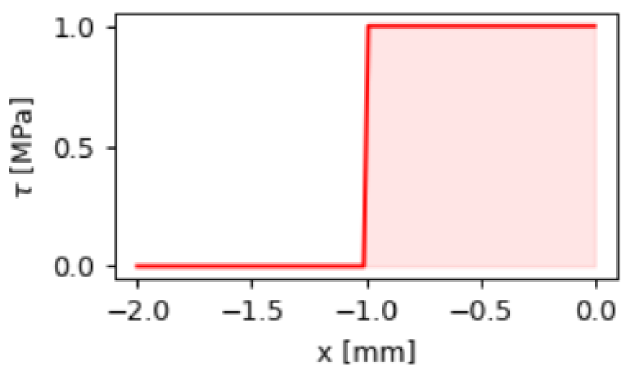

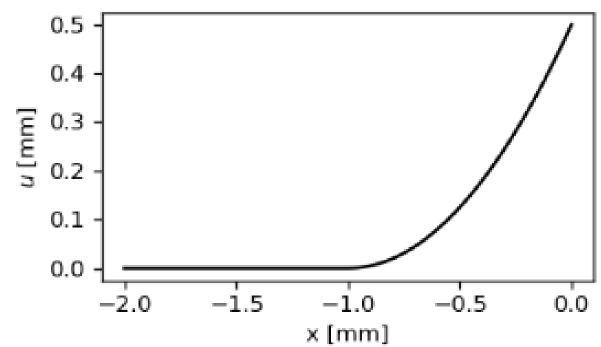

2. Dissipation and storage processes spatially separable?

$$W = \int_0^w P(w) \, \mathrm{d}w \qquad U_\omega = \frac{1}{2} P w \qquad U_\pi = \frac{P^2}{2K_0} \qquad G = W - U \qquad U = \int_{\Omega_{\mathrm{el}}} \sigma(x) \varepsilon(x) \, \mathrm{d}x$$

Dissipated energy → it tends to localize into a small material volume

Realize:

The example combined two energetic devices
one fully dissipative device → no energy storage (ideally plastic bond), and
one non-dissipative device → perfect energy storage (elastic bar)

Idea:

If it is possible to integrate the stored energy in all material points over the domain can we also integrate the released energy over the domain?

$$G = \int_{\Omega_{\mathrm{diss}}} \tau(s,x)s(x)\,\mathrm{d}x = \int_0^w P(w)\,\mathrm{d}w - \int_{\Omega_{\mathrm{el}}} E\varepsilon(x)\varepsilon(x)\,\mathrm{d}x$$

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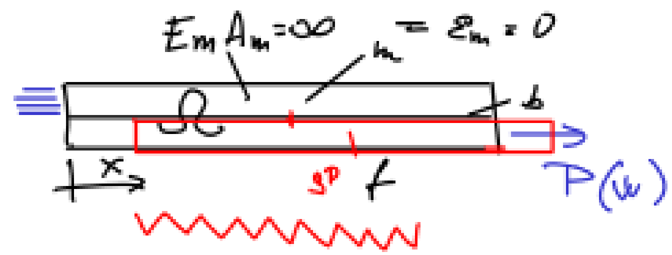

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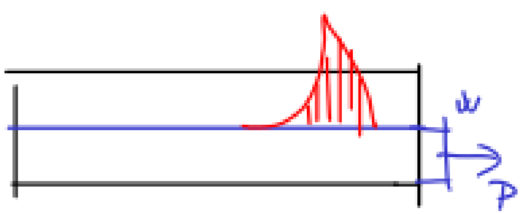

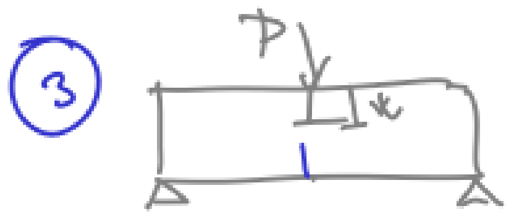

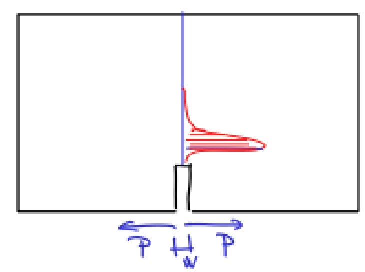

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This is really useful! We can evaluate dissipated energy locally!

If we know where the localization and crack propagation occurs,
we can focus just onto this region and ignore the non-dissipative
(elastic/unloading) rest of the domain

→ this is the fundamental concept behind methods describing **fracture**

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