

## Nonlinear bond slip behavior modeled using plasticity

Assume that a bond behavior is governed purely by plasticity model with the following parameters:

$$E_b = 100 \text{ MPa/mm}, \tau_0 = 10 \text{ MPa}, K = 10 \text{ MPa/mm}, \gamma = 5 \text{ MPa/mm}$$

- a) Plot the bond-slip law for a monotonic loading up to  $s = 1.0 \text{ mm}$ ,  
and determine the slope of the hardening range.
- b) Calculate the total slip at the state with the bond stress  $\tau = 15.0 \text{ MPa}$ .
- c) Calculate the plastic slip at the states with the total slip  $s = 0.1 \text{ mm}$ , and  $s = 0.6 \text{ mm}$ .

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and determine the slope of the hardening range.

## Solution:

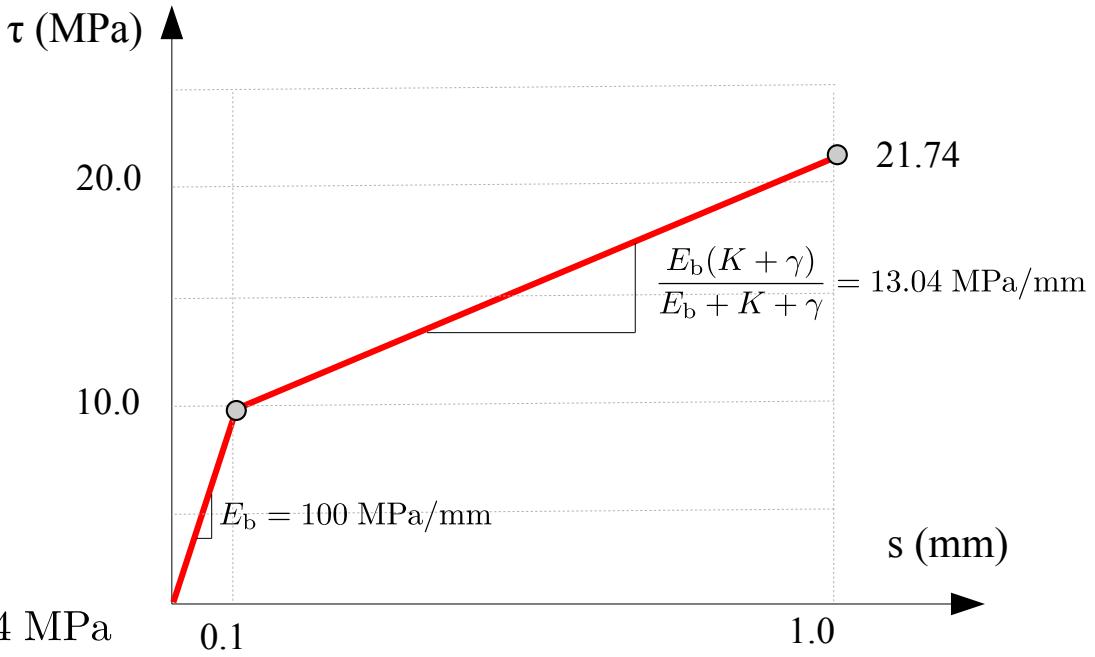
The slope of the hardening range

$$\frac{E_b(K + \gamma)}{E_b + K + \gamma} = \frac{100(10 + 5)}{100 + 10 + 5} = 13.04 \text{ MPa/mm}$$

The bond stress at  $s = 1.0 \text{ mm}$

$$\tau(s = 1.0) = \tau_0 + \frac{E_b(K + \gamma)}{E_b + K + \gamma} \times (1.0 - s_0)$$

$$\tau(s = 1.0) = 10 + \frac{100(10 + 5)}{100 + 10 + 5} \times (1.0 - 0.1) = 21.74 \text{ MPa}$$



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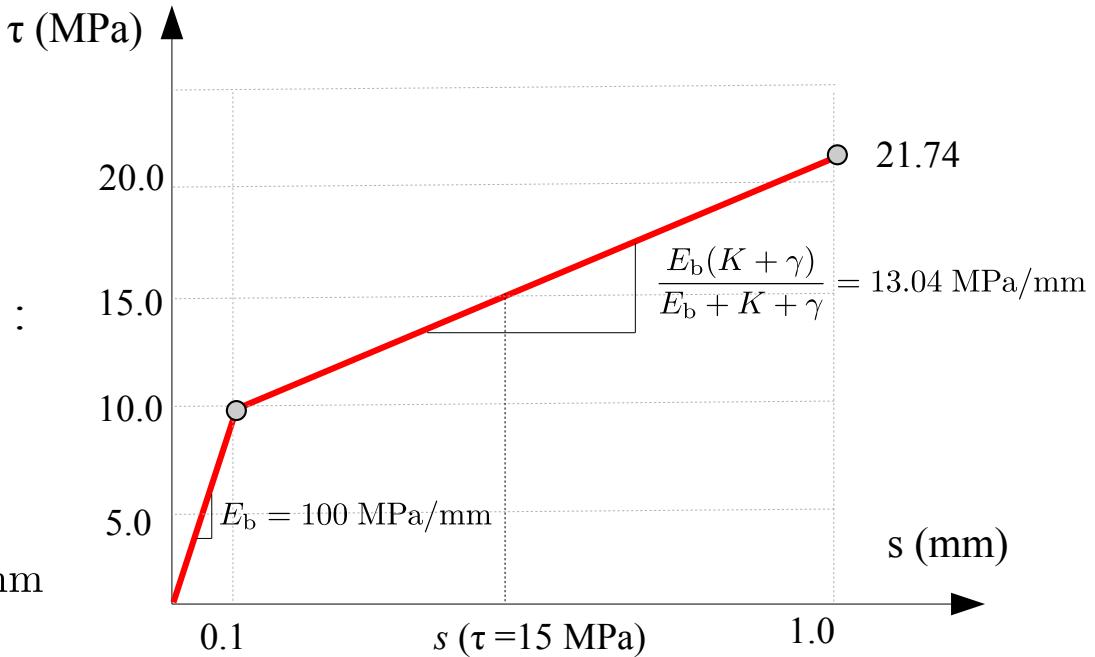
The bond stress in the range  $s \geq s_0$ :

$$\tau(s) = \tau_0 + \frac{E_b(K + \gamma)}{E_b + K + \gamma} \times (s - s_0)$$

The total slip for a given bond stress in the range  $s \geq s_0$ :

$$s(\tau) = s_0 + (\tau - \tau_0) \times \frac{E_b + K + \gamma}{E_b(K + \gamma)}$$

$$s(\tau = 15) = 0.1 + (15 - 10) \times \frac{100 + 10 + 5}{100(10 + 5)} = 0.483 \text{ mm}$$



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c) Calculate the plastic slip at the states with the total slip  $s = 0.1 \text{ mm}$ , and  $s = 0.6 \text{ mm}$ .

## Solution:

- at the slip  $s = 0.1 \rightarrow$  The plastic slip  $s^p = 0.0$
- at the slip  $s = 0.6$

$$\tau(s) = \tau_0 + \frac{E_b(K + \gamma)}{E_b + K + \gamma} \times (s - s_0) = 16.52 \text{ MPa}$$

$$s^e(s = 0.6) = \frac{\tau(s = 0.6)}{E_b} = \frac{16.52}{100} = 0.165 \text{ mm}$$

$$s^p(s = 0.6) = s - s^e(s = 0.6) \\ = 0.6 - 0.165 = 0.435 \text{ mm}$$

